Primer *of* Celestial Navigation

By John Favill, M.D.
Associate Member, U. S. Naval Institute
Navigator, U. S. Power Squadrons
Colonel, Inactive Reserve, U. S. Army

Third Edition
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Contents

Part III: Supplementary

List of Illustrations

List of Tables

Preface to Third Edition

PEARL HARBOR and all its consequences have come since this Primer's first birthday. Thousands of officer candidates have taken up with grim determination the study which to me for years has been a delightful hobby.

In preparing a new edition, the chief problem was whether to cut to the bone and leave only the bare essentials for practical purposes or to retain what seemed, and still seems to me, essential for a secure understanding of the subject.

Pruned down to lines of position only and plotting on charts or plotting sheets, it is possible to get along without the following:

Meridian altitudes, reductions to the meridian and ex-meridian sights for latitude, interval to noon, time of transit, time of sun on the prime vertical, time-sights for longitude, sights for determination of chronometer error, trigonometry, logarithms, much of the sailings, basic astronomy, and sidereal time. (I realize one need not even know the meaning of declination in order to use the almanac and H. O. 214 effectively.)

The original purpose of the Primer however was to help clarify obscurities. It seems to me this can best be done by a build-up from simple to complex, and by giving some procedures which, though no longer needed, have been strong links in the chain leading to present practice. So the main framework of this book remains unchanged.

Some omissions and many more additions will, I hope, increase usefulness. While written primarily for surface work, the principles here given are, of course, basic for aerial navigation. No attempt is made to supply detail for the latter, which is so adequately covered in Dutton.

I am still convinced that years of custom have made it more natural and easy to look, in imagination, at this world and the universe from above the north rather than from below the south pole. Hence I have retained the style of diagram in general use until the years 1936-39, when the leading texts shifted their viewpoint to face Antarctica. Probably the important thing is to get the habit of using some time diagram—not a particular type.

In this third edition, about twenty-five minor corrections, changes or insertions have been made without change of page number. An important addition on Sidereal Time has been added on page 249 and the author's new "Uniform Method for More Exact Time of Local Transit of Any Body" will be found on pages 254–258.

Having been found physically ineligible for active duty, I launch this new Primer with a bold hope that somehow its influence, however small, will count in the score of ultimate Victory.

J. F.

February, 1944
Chicago, Illinois

Introduction

RESCUE AT SEA of human beings from a ship in distress is usually the result of a radio message. One or more vessels respond by hastening to the locality of the trouble. Few of us on shore, reading the press accounts of such a rescue, fail to feel thrilled that the genius of Marconi has again added to the large total of lives saved in this way. But I wonder how often it is realized that many centuries of development of the art and science of navigation led up to the ability of the master to state the position of his ship on this globe within a radius of about one mile. Without this ability, there would be little help in wireless.

The term *Navigation* covers a number of items. It is broadly divided into (1) Geo-navigation and (2) Celo-navigation. *Geo-navigation* includes the methods of locating the ship's position by earth landmarks or characteristics. It is subdivided into (a) *Piloting,* which has to do with bearings, buoys, lighthouses, soundings, radio beams and chart study and (b) *Dead Reckoning* which deals with methods of estimating the distance covered and point reached in a given interval by means of compass observations, log readings, record of engine revolutions and a few calculations. *Celo-navigation* or *Celestial Navigation* is also called *Nautical Astronomy*. It is the subject concerned with position finding away from all landmarks when a ship is at sea. This is done by sextant observations of the sun or moon or certain planets or stars, with notation of

exact time of each observation and with the aid of the *Nautical Almanac* and certain tables.

The professional seaman learns these things as part of his job and probably gets very tired of their routine practice. The occasional cruising yachtsman or even he who seldom leaves land can find the study of navigation a very fascinating one and may even let it become an absorbing hobby. Such at any rate has been my happy experience.

Some years ago on Cape Cod a friend who had a sextant helped me take an observation of the sun and worked out the longitude via Bowditch. I tried to understand the various steps but could not. Buying a Bowditch I began to study these matters from the ground up. Many more books were soon accumulated. A sextant was purchased and sights were taken over Lake Michigan and worked out, with gradually increasing accuracy.

Some of the by-products of this activity have been: an appreciation of the ingenious powers of spherical trigonometry; an interest in astronomy with realization for the first time that there actually is some reason, for one who is not an astronomer, to know certain of the stars; some understanding of the various kinds of time-keeping; a growing taste for sea stories and sea-lore in general; enjoyment of that fine magazine, *U. S. Naval Institute Proceedings*, and the popular yachting monthlies; the fun of having a paper on a small overlooked point accepted and published; correspondence with several men in different continents as a result of this; the diversion of working out text-book problems; glimpses of the history of navigation; and, probably most important of all, the

mental refreshment of studying and doing something utterly different from one's professional work.

The following compilation is limited to off-shore position finding and so omits all consideration of the subject of Piloting. It is intended in no way to supplant for the beginner such splendid texts as Bowditch or Dutton but rather to smooth the road to those books, to which frequent reference will be found. I have used new ways in presenting some of the old subjects. But my main purpose is to prevent certain confusions which come to the amateur when first attempting this study. The best-known books are written by men of much learning who often seem to have forgotten certain stumbling blocks which they themselves passed long ago. I have done some of my stumbling quite recently and am now trying to show how such can be prevented. The student who wishes to learn a method as quickly as possible and who does not have internal distress at following rules blindly and knowing nothing of their origins will have no need of this manual. But those who enjoy understanding why things are done and what they mean, and who like to have all important steps included will, I hope, find in these pages a certain satisfaction.

J. F.

Part I: Fundamentals

1. Astronomical

THE PRACTICAL NAVIGATOR makes use of certain of the so-called heavenly bodies to locate his position. These are the sun, the moon, four planets (Venus, Mars, Jupiter, Saturn), and fifty-five stars.

It is vital for a clear understanding of the uses of these bodies to consider their positions in space and to orient ourselves. A long story in the history of science lies behind our present knowledge.

The earth was known to be a sphere in the fourth century B. C.

Eudoxus of Cnidos (about 360 B. C.) taught that the sun, moon and planets all moved around the earth, which was stationary.

Aristarchus of Samos (310-230 B. C.) considered that the sun and stars were stationary and that the earth revolved around the sun.

Hipparchus (130 B. C.) preferred and developed the geocentric system of Eudoxus. However, he invented plane and spherical trigonometry and observed the phenomenon known as precession, to be described later. He was the first of the Greeks to divide the circle into 360 degrees and attempted to determine the positions of places on the earth by measuring their latitude and longitude.

Ptolemy of Alexandria (127-151 A. D.) expounded the

TABLE 1

THE SOLAR SYSTEM

	Symbol	Diameter in Miles	Distance from Sun in Millions of Miles	Period of Revolution	Satellites	Apparent Magnitude
Sun..........	⊙	864,000	0	0	10	−27
Mercury.......	☿	3,106	36	88 d.	0	−1 to +3
Venus.........	♀	7,705	67	225 d.	0	−4 to −3
Earth.........	⊕	7,918	93	365 d.	1	
Mars.........	♂	4,207	141	687 d.	2	−3 to +2
Asteroids......		480−	332	3–9 yrs.	0	+6 and up
Jupiter........	♃	86,718	483	12 yrs.	11	−2
Saturn........	♄	71,520	886	30 yrs.	10	0
Uranus........	♅	31,690	1,782	84 yrs.	4	+6
Neptune.......	♆	31,069	2,792	164 yrs.	1	+8
Pluto.........	♇	3,600	3,700	250 yrs.	0	+14

geocentric system so well that it acquired his name and was held by scholars for fourteen centuries.

Copernicus (1473-1543) wrote a great book developing the ideas of Aristarchus, then eighteen centuries old, that the sun was the center of our system, and stationary, with the earth and planets revolving around it. This book was published only in time for Copernicus to see a copy on his deathbed.

Tycho Brahe (1546-1601) observed the planetary motions with improved instruments and recorded a great number over many years.

Johann Kepler (1571-1630) after years of calculations, using much of Tycho's data, found the three laws of planetary motion that bear his name.

Galileo Galilei (1564-1642) was the pioneer of modern physics. He made the first telescope, discovered the moons of Jupiter, and proved the conclusions of Copernicus by actual observations.

Isaac Newton (1642-1727) worked out the laws of motion and gravity. His *Principia* published in 1687 "marks perhaps the greatest event in the history of science" (Dampier-Whetham).

An *ellipse* is a closed curve such that the sum of the two distances from any point on its circumference to two points within, called the *foci*, is always constant and equal to the major axis of the ellipse.

The solar system consists of a group of bodies all revolving around the sun. They go counterclockwise if seen from above the north pole, in ellipses—not circles—with the sun at one of the foci of each ellipse. (*See Table* 1.)

Perihelion is the point on a planet's orbit nearest the sun. It is reached about January 3 for the earth (91,500,000 miles). The earth moves faster along its orbit when nearer the sun.

Aphelion is the point on a planet's orbit farthest from the sun. It is reached about July 3 for the earth (94,500,000 miles).

The so-called "fixed stars" lie far away from our solar system, so far in fact that only very slight shifts can be found in the positions of the nearer ones when we observe them from opposite ends of the earth's orbit. Their own actual motions appear negligible. The nearest fixed star is Proxima Centauri, 4⅓ light years away or 26,000,000,000,000 miles. Deneb, a bright star often used in navigation, lies 650 light years away or 3,900,000,000,000,000 miles!

Brightness of bodies is recorded by numbering the *apparent magnitude*. From minus quantities through zero into plus quantities represents diminishing brightness. Plus sixth magnitude is just visible to the naked eye. Each magnitude is 2½ times brighter than the next fainter one. The brightest heavenly bodies are shown in Table 2.

The *Nautical Almanac* gives the necessary data for the exact location of 55 of the brightest stars and less full data for an additional list of 110 other stars for occasional use.

The earth rotates on its axis daily as well as revolving in its orbit around the sun in a year. The earth's axis of rotation is inclined to the plane of the earth's orbit at about 66½ degrees and the north pole points approximately toward the north star or Polaris. The plane of the

earth's equator is therefore inclined to the plane of the orbit by about 23½ degrees.

TABLE 2

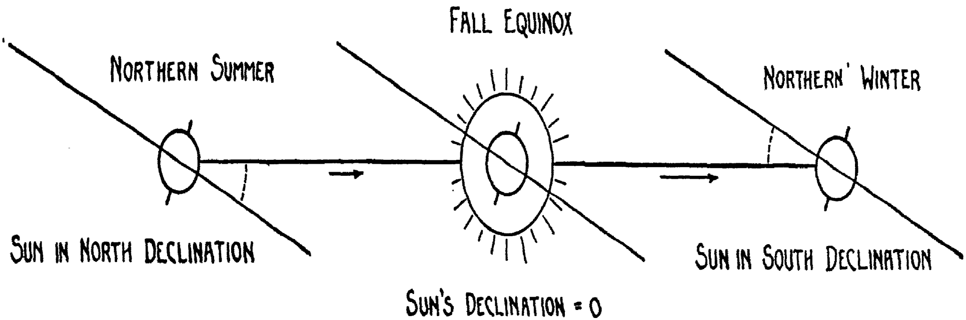
THE BRIGHTEST HEAVENLY BODIES

Sun	—27	Achernar	+1
Moon (full)	—13	β Centauri	+1
Venus	— 4 to —3	Altair	+1
Mars	— 3 to +2	Acrux	+1
Jupiter	— 2	Aldebaran	+1
Sirius	— 2	Antares	+1
Canopus	— 1	Betelgeux	+1
Saturn	0 to +1	Pollux	+1
Vega	0	Spica	+1
Arcturus	0	Deneb	+1
Capella	0	Fomalhaut	+1
Rigel	0	Regulus	+1
Rigel Kentaurus	0	β Crucis	+1
Procyon	0	γ Crucis	+2

A *meridian* is an imaginary line on one half of the earth, formed by the intersection with the earth's surface of a plane which passes through both poles and is therefore perpendicular to the equator.

The *longitude* of a place on the earth is the arc of the equator intercepted between the place's meridian and the Greenwich, England, meridian, being reckoned east or west from Greenwich to 180°.

The *latitude* of a place on the earth is the arc of the place's meridian intercepted between the equator and the place, being reckoned north or south from the equator to 90°.



HORIZONTAL LINE REPRESENTS THE PLANE OF THE EARTH'S ORBIT SEEN ON EDGE.
OBLIQUE LINES REPRESENT THE PLANE OF THE EARTH'S EQUATOR, SEEN ON EDGE, AT THREE POINTS IN EARTHS ORBIT.
ARROWS SHOW EARTH'S PROGRESS FROM SUMMER THRU FALL TO WINTER (N. HEMISPHERE).

Fig. 1. Declination.

A *parallel* of latitude is an imaginary circle formed by the intersection with the earth's surface of a plane passed parallel to the plane of the equator.

Declination is one of the most important things to understand in navigation. It is the angle that a line from the center of the earth to a given heavenly body makes with the plane of the earth's equator. Remember it has nothing to do with where you are on the earth. It is described as north or south in reference to the plane of the equator. The declinations of the stars change very little in the course of a year. There are great changes however in the declinations of the sun, moon, and planets.

Equinoxes. As the earth travels around the sun from winter to summer, the (extended) plane of the equator approaches nearer and nearer the sun finally cutting its center at an *instant* known as the Spring or Vernal equinox on March 21. The plane passes through the sun and beyond till the earth turns back in its swing around the sun when the process is repeated through the Fall or Autumnal equinox on September 23 and so on to winter. The days and nights are equal at the equinoxes except at the poles. The sun's declination is then zero and it "rises" in exact east and "sets" in exact west. (*See* FIG. 1.)

Solstices. There are two instants in the earth's journey around its orbit when the sun attains its highest declination north or south. They are designated Summer solstice and Winter solstice according to the season in the northern hemisphere, and occur June 21 and December 22, respectively.

Celestial Sphere. Declination alone is not enough to locate a body in the sky. We require some means of regis-

tering its east or west position as well as its north or south. For this purpose we imagine a great hollow "celestial" sphere to lie outside our universe with the earth at its center. The heavenly bodies can be projected onto the inner surface of this sphere as can also the plane of the earth's equator, or any meridian or point on the earth as though observed from the earth's center. Likewise the position of the sun's center at the time of the Vernal equinox on March 21 as seen from the earth can be projected. This *point* is also called the Vernal equinox or "First Point of Aries." Its symbol is ♈. It is taken as the zero of measurement around the *celestial equator* or *equinoctial* which is the projection of the earth's equator. The extension of the plane of the earth's orbit to the celestial sphere produces a circle on the latter known as the *ecliptic.* The two points where the equinoctial and ecliptic intersect mark the two equinoxes. The angle of about 23½° of these intersections measures the *obliquity of the ecliptic.*

Right Ascension is a measure of angular distance around the celestial equator, eastward from ♈. It is expressed in hours (and minutes and seconds) up to 24. By giving a body's declination and right ascension, we pin it down to a definite location on the celestial sphere just as a place on earth is fixed by giving its latitude and longitude. The declination corresponds to the parallel of latitude and the right ascension to the meridian of longitude. We will see later that the R. A. of the projected local meridian equals the local sidereal time and 24 hours of sidereal time measure practically one exact rotation of the earth.

Translating time to arc we have:

24 h =	360°
1 h =	15°
4 m =	1°
1 m =	15′
4 s =	1′
1 s =	15″

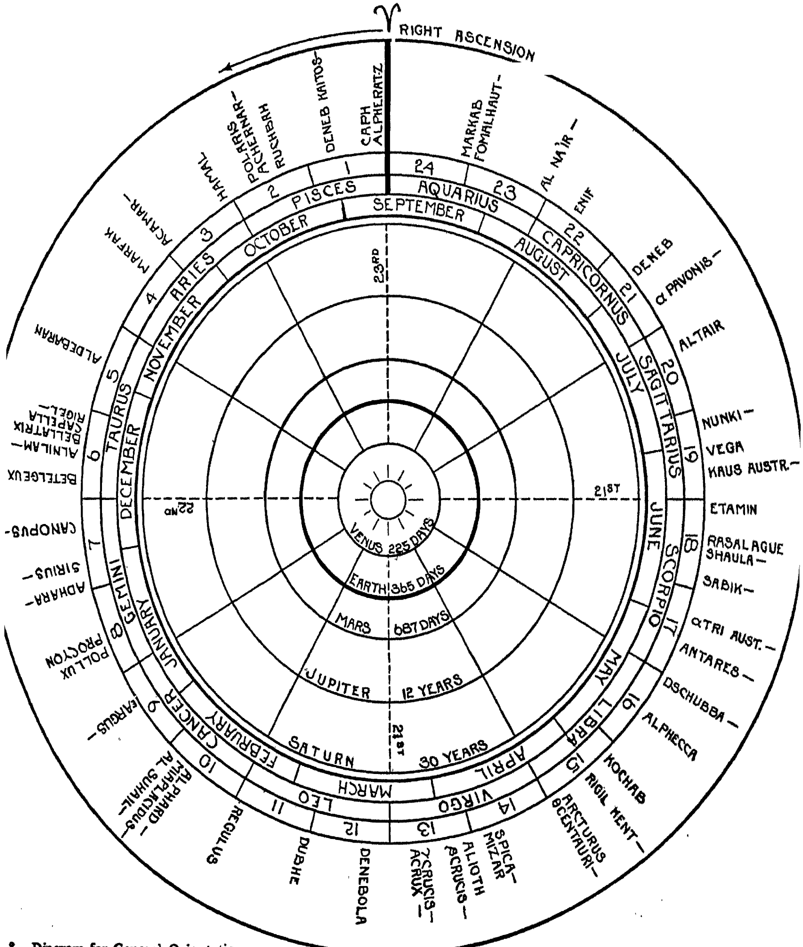
Speaking in terms of apparent motion, we could say a body's R. A. is the distance (angle or hour) at which it is trailing the ♈ in the latter's journey around the equinoctial.

The importance of correctly orienting ourselves in relation to solar system and stars and their real and apparent motions and our real motion cannot be overstated. Much of the bewilderment of the beginner comes from encountering emphasis on apparent motion with inadequate explanation of the real situation. For instance, "the sun's path among the stars" is a most confusing expression. It would be plain if we could see stars in the daytime and would compare the sun's position at a certain time on two successive days. Then we would observe the shift to the eastward which is meant. It is just as though we were in a train going forward and, looking out a window on the left side, observed a tree 200 yards away and a bit of woods 2,000 yards away and immediately shut our eyes (passage of 24 h) and quickly looked again. The nearby tree (sun) then seems to have moved to the left in relation

to the distant trees (stars). Another expression, the "revolving dome" of the celestial sphere, is also misleading. Remember our earth is rotating to the east. The celestial sphere therefore only *seems* to rotate to the west. In north latitudes we see the north star as though it were a pin or hub on the inside of a sphere which turns around it counterclockwise. Bodies are swinging over it to the left and returning under it to the right.

Another extremely important fact to realize is this: the earth because of its progress around its orbit must make a little more than one exact rotation between two successive noons. From Monday, at the instant the sun is due south in the northern hemisphere, till Tuesday, when the sun is again due south, there has been something over one complete rotation of the earth. It is incorrect to explain this by saying the sun has shifted somewhat to the east. The truth is that the progress of the earth in its orbit at a speed of 30 kilometers per second has altered the direction of the earth from the sun. This makes the sun seen from the earth appear to have shifted eastward. This will explain why a true sun day is a longer bit of time than a star day which requires practically only one exact rotation. (*See* FIG. 2.) Owing to the eccentricity of the earth's orbit and obliquity of the ecliptic these sun days are not exactly equal.

At this point the student may profit by inspection of Figure 3, a diagram designed to make more clear some of the matters so far discussed. The following explanation of the diagram should be carefully studied.



3. Diagram for General Orientation.

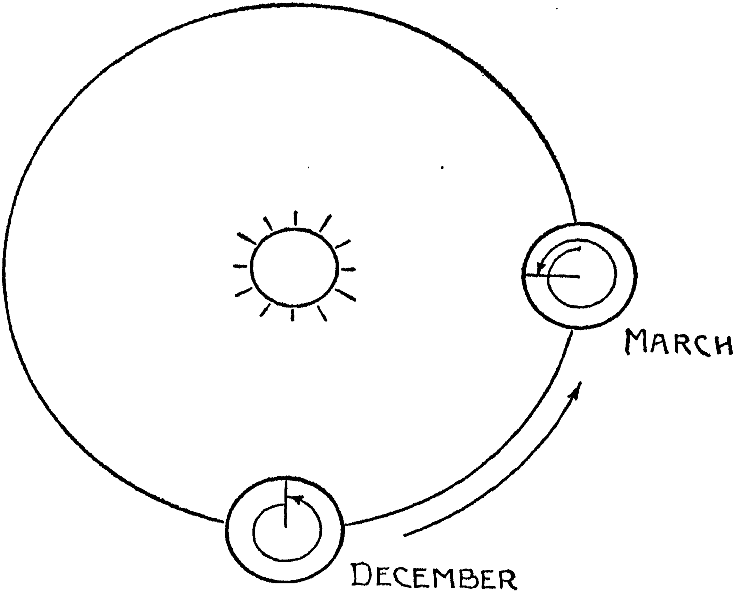


FIG. 2. Excess of One Rotation.

Earth's gain of ¼ of a rotation when ¼ around Orbit. Line on Earth: Meridian of some place, say Greenwich, at Noon in each instance.

Explanation of Diagram

Outer circumference represents the celestial sphere in the plane of its equator seen from north.

♈ at top is "First Point of Aries" from which right ascension is counted in a circle divided into 24 hours, numbered counter-

clockwise. These are shown in the 2nd band. Right ascension on the celestial sphere is similar to longitude on the earth. Declination on the celestial sphere corresponds to latitude on the earth but cannot be shown here in the one plane.

Fifty-five navigational stars including Polaris whose right ascensions and declinations change very little are shown in the outer band in approximately correct positions. Those with a minus sign added are of southern declination and lie below (behind) the diagram while all others are of northern declination above (in front of) the diagram. These are the stars for which complete data are given in the *Nautical Almanac* and which are most easily used for position finding.

The 12 "Signs of the Zodiac" are included in the 3rd band for popular interest only. They are named from various constellations of stars which lie in a belt not over 8° above and below the plane of the earth's orbit, or the ecliptic on the celestial sphere. Because of the slow swing of the north end of the earth's axis in a circle around the north pole of the ecliptic, clockwise if seen from above, once in 26,000 years, the equinoxes (points) gradually shift to the westward. Hence the Spring or Vernal equinox (♈), which when named about 2,100 years ago was actually in the sign of Aries, is now almost through Pisces. Note that this point ♈ on the celestial sphere is where the sun seems to be when seen from the earth on the 21st of March. At that time (also known as the Spring or Vernal equinox) the sun's declination is changing from south to north. Dotted lines show this date and also the dates of the Fall equinox, the Summer solstice and the Winter solstice.

The 4th band of months is for locating the earth in its orbit.

The five circles outside the sun as labeled are for the orbits of the earth and the four planets used in navigation, with their periods of rotation, all counterclockwise.

The "spokes" are to show two-hourly intervals of R. A.

How to Use Diagram

Make a dot on earth's orbit corresponding to the date.

Look in N. A. for R. A. of each of the planets for the same date and make a dot on each orbital circle at the given R. A.

Look in N. A. for moon's R. A. for the same date and time and make a dot near earth corresponding to this.

Turn diagram till sun and earth are in a line "across the page" with sun on the right.

Lay a ruler across diagram edge up so edge passes through sun and earth.

All *above* ruler edge represents stars and planets available around evening twilight of day in question at equator.

Holding diagram stationary, rotate ruler edge counterclockwise with earth as center, through 90°. All to *left* of ruler edge represents sky of midnight at equator.

Similarly rotate ruler edge counterclockwise with earth as center through another 90°. All *below* ruler edge represents sky of twilight next morning at equator.

If position of observer were at one of the poles, all stars and planets with the corresponding declination would be visible during the six months of darkness while those of opposite declination would be invisible.

As position of observer increases in latitude from equator to, say, the north pole, the visible stars of southern declination will diminish in number, beginning with the highest declinations, while the visible stars of northern declination will increase in number, also beginning with the highest declinations.

Note: This diagram is of course not to scale and only an approximation. Remember the planet dots do not represent exact positions of planets in their orbits but rather the planets' positions on the celestial sphere as seen from earth and expressed in R. A. The earth's journey around its orbit accordingly results in apparent retrograde movement of planets at times. (Jupiter, 1937: Jan. 1, R. A. $18^h\ 29^m$; May 1, R. A. $19^h\ 56^m$; October 1, R. A. *$19^h\ 18^m$*; December 1, R. A. $19^h\ 52^m$.)

The orbits are in reality ellipses with the sun at one focus and not circles as here given.

The plane of the earth's orbit, near which the Zodiac signs are grouped, and the plane of the earth's equator, along which the R. A. hours are measured, actually lie 23½ degrees apart but are here supposed to be compressed into the plane of the diagram. Likewise the planes of the planets' orbits are leveled into the diagram. The months are here marked as though of uniform length. Periods of rotation are approximate.

In order to show how far above and below the previous diagram the various stars lie, a list of declinations is provided. It shows the order the stars would be met with from the north pole to the south pole of the celestial sphere (*See Table* 3.)

A chart in the back of the *Nautical Almanac* combines data from the preceding diagram and list (R. A. and Dec.) in one plane. It is of the Mercator type (which will be explained later) and so is somewhat confusing to the beginner.

A celestial globe is a great help in learning star locations, constellations, etc. The only difficulty is that it shows the celestial sphere *from without* and one must always imagine a given group as seen from the center of the globe in order to duplicate the actual group in the sky.

The moon's motion and "phases" deserve some attention here. The moon is about 238,840 miles away. It rotates on its axis only once, counterclockwise from above, in its trip of revolution around the earth; hence it always keeps the same face toward us. It makes a complete revolution judged by its relation to stars in 27⅓ days but to completely circle the earth (which is traveling in its orbit) requires 29½ days. It is cold and only shines by light from the sun. It revolves around the earth counterclock-

wise seen from above the north pole, or from west to east. As this motion is so much slower than the earth's daily rotation, the moon seems to be going each night from east to west. Each successive night, however, at a given time, its position is about $12\frac{1}{2}°$ farther east. After reaching

TABLE 3

DECLINATIONS OF NAVIGATIONAL STARS

NORTH		SOUTH	
88°	Polaris	1°	Alnilam
74	Kochab	8	Rigel
62	Dubhe	8	Alphard
59	Ruchbah	10	Spica
58	Caph	15	Sabik
56	Alioth	16	Sirius
55	Mizar	18	Deneb Kaitos
51	Etamin	22	Dschubba
49	Marfak	26	Antares
45	Capella	26	Nunki
45	Deneb	28	Adhara
38	Vega	29	Fomalhaut
28	Alpheratz	34	Kaus Australis
28	Pollux	36	θ Centauri
26	Alphecca	37	Shaula
23	Hamal	40	Acamar
19	Arcturus	43	Al Suhail
16	Aldebaran	47	Al Na'ir
14	Denebola	52	Canopus
14	Markab	56	γ Crucis
12	Rasalague	56	α Pavonis
12	Regulus	57	Achernar
9	Enif	59	ε Argus
8	Altair	59	β Crucis
7	Betelgeux	60	Rigil Kentaurus
6	Bellatrix	62	Acrux
5	Procyon	68	α Tri. Australis
		69	Miaplacidus

"full" it "rises" about 50 minutes later each evening. Previous to full it may be seen as early as mid-afternoon and subsequent to full it may be seen up to several hours after sunrise or even till noon.

As the plane of the moon's orbit is near the plane of the earth's, the moon will appear to observers in the northern hemisphere lower in the southern sky in summer and higher

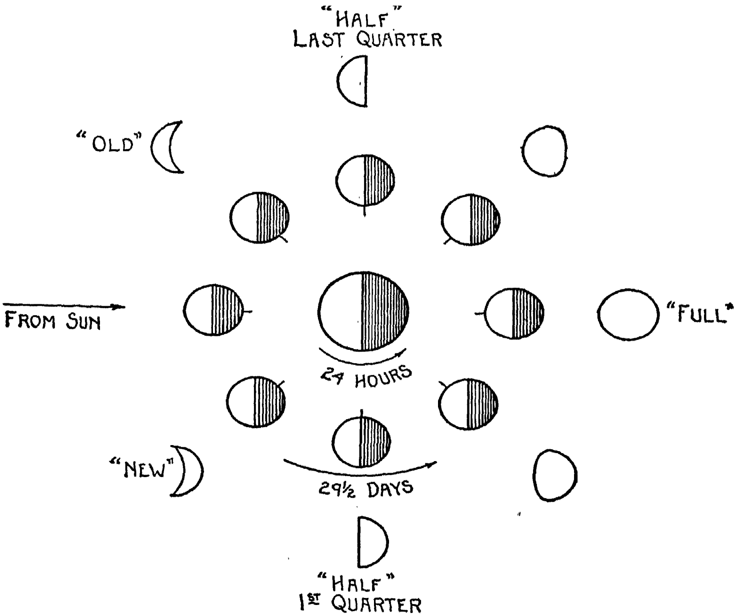


FIG. 4. Phases of the Moon.

Diagram is from above North Pole of Earth. Outer row shows how each position appears in our North Latitudes. The small projection from the Moon is a fixed point and shows that the same face remains toward the Earth.

in winter. This will be understood if Figure 1 is now reviewed. Figure 4 will explain the moon's phases.

Miscellaneous Facts

Precession of the Equinoxes. Each year the equinoxes (Spring and Fall) come about 20 minutes sooner. An equinox can either be thought of as a certain instant in the earth's journey around its orbit when the plane of the equator cuts the center of the sun, or as the point on the celestial sphere where the sun appears to be at that time, as it is changing from S. to N. or from N. to S. declination. These points are shifting to the west, or in a direction opposite to the earth's orbital motion. They do so about 50″ of arc on the celestial sphere per year.

This all happens because the extended ends of the earth's axis are very slowly making circular motions around the poles of the ecliptic. (Disregard the ellipses which the axis makes in one year because at the distance of the celestial sphere these ellipses would be extremely small.) The projection of the earth's north pole describes a circle with a radius of about 23½° around the north pole of the ecliptic, counterclockwise as we look up at it, clockwise as seen from outside celestial sphere looking down, once in 26,000 years. The plane of the earth's equator, perpendicular to the axis, must likewise shift and, when meeting the sun's center at equinox, will be intersecting the track of the earth's orbit on each side at a point slightly more westward. This may be represented (*see* FIG. 5) by a metal circular ring (plane of earth's orbit) with a disc (plane of earth's equator) slightly

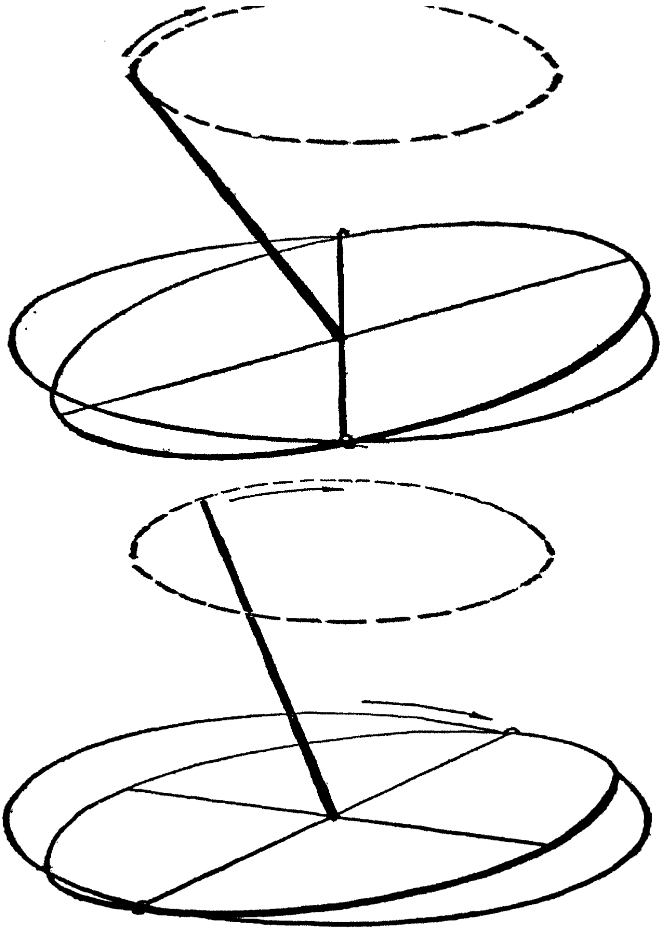


FIG. 5. Precession of the Equinoxes.

smaller inside the ring and loosely attached at two opposite points (equinoxes) and inclined 23½° to the ring (obliquity of ecliptic). A rod projects up from the disc's center (parallel to earth's axis). Shifting the attachments clockwise when looking down at disc will show rod's tip describing a circle clockwise around a point above center of ring (north pole of ecliptic). The rod is placed in center instead of at edge by earth's orbit because it shows the effect more clearly and because the misplacement is negligible when considering the extreme distance of the celestial north pole.

The Vernal equinox (♈) called "First Point of Aries" was in that constellation when named about 2,100 years ago but has now shifted westward almost through the next constellation "Pisces."

As a result of this performance, there is a succession of stars called north stars through the centuries. Our present one, Polaris, is therefore only playing a temporary role. The series is as follows:

Vega	12,000 B. C.
ρ Hercules	7,200
Thuban	3,000
Polaris	2,100 A. D.
Er Rai	4,200
Alderamin	7,500
δ Cygni	11,500
Vega	14,000
etc.	

Another result of precession is that there is a slow increase in the right ascensions of all stars. Naturally, if the

starting point for measurement (♈) is moving west and the measurements are made to the east, these measurements will grow larger.

The north pole of the ecliptic is at R. A. 18, Dec. 66½ N. and the south pole at R. A. 6, Dec. 66½ S.

It may be wondered why the hottest part of northern summer is not halfway between Spring and Fall equinox at June 21 and the coldest part of winter at December 22. The explanation is probably that extra time is required to warm up the earth in summer and to cool it off as winter approaches.

Earth's perihelion, the position nearest sun, occurring in northern winter and southern summer, and aphelion, the position farthest from sun, occurring in northern summer and southern winter, might lead one to expect the southern hemisphere to show more extremes of climate, hotter in summer and colder in winter. However, the eccentricity of the orbit is so slight that no great difference is noted. The eccentricity of an ellipse is expressed by the following ratio:

$$\frac{\text{Distance from center to one focus}}{\text{Distance from center to one end of major axis}}$$

For the earth's orbit, this is only about $1/60$, so the orbit is not far from circular. Nevertheless, it is 186 days from Spring to Fall equinox, and only 179 days from Fall to Spring equinox. The sun therefore is in that focus which is nearer to us in December and farther from us in June. (*See* FIG. 6.)

All the planet's orbits lie within 8° of the ecliptic and the four navigational planets lie within 3°.

Prof. Dayton C. Miller reported (*Science*, June 16, 1933)

after very exhaustive observations on the speed of light, that the entire solar system was moving as a body through space at a speed of 208 kilometers per second toward a point in R. A. 4^h 56^m, Dec. 70° 30′ S. This is close to the south pole of the ecliptic and about 20° south of the second brightest star, Canopus.

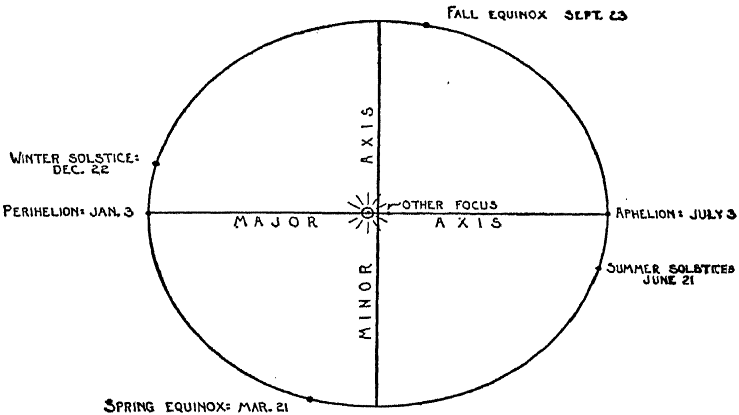


FIG. 6. The Earth's Orbit. From Above. (True eccentricity is even less than here.)

Kepler's Laws are as follows:

1. The orbit of every planet is an ellipse, having the sun at one of its foci.
2. If a line is supposed to be drawn from the sun to any planet, this line passes over equal areas in equal times.
3. The squares of the times of revolution about the sun of any two planets are proportional to the cubes of their mean distances from the sun.

2. Time

THE SUBJECT OF TIME is one of the tough spots in the study of navigation. This is partly because new ways of thinking about time are required and partly because the explanations are often inadequate. We will try now to proceed carefully and put in all the steps so that there may be no misunderstanding.

Apparent Time. We think of time as somehow measured by the apparent movement of the sun each 24 hour day around the earth, caused of course by the earth's rotation with the sun stationary.

We have seen in Chapter 1 that, while the earth rotates on its axis at a uniform speed, it does not travel in its orbit around the sun at a uniform speed. Its speed is faster when closer to the sun in the elliptical orbit. We also saw that, because of progress along its orbit, the earth must make somewhat more than a complete rotation on its axis in order to bring the sun from a meridian one day back to the same meridian the next day. Finally, we noted the obliquity of the ecliptic.

So days measured by sun noons are not uniform because:

(a) The varying speed of the earth in its orbit varies the necessary daily excess over one revolution, and,

(b) As the plane of the earth's orbit in which the sun appears to move, does not coincide with the plane of the earth's equator along which movement is measured, equal divisions of the former do not make equal divisions of the latter when projected onto it.

No clock can be made to keep step with this true solar or, as it is called in navigation, apparent time. Its day begins at midnight. Greenwich (England) apparent time and local apparent time are designated respectively G. A. T. and L. A. T.

Mean Time. An imaginary or "mean" sun is therefore utilized which crosses a given meridian at uniform intervals throughout the year. The interval is the average of the true sun days. The mean sun is thought of as in the plane of the earth's equator all the time. The total of days in the year is the same as for apparent time. Days are divided into 24 hours of 60 minutes each and each minute into 60 seconds. This is the time our regular clocks keep. In navigation, the hours are numbered from 0 to 24. Ship chronometers are set to keep Greenwich time and almanacs give data for this. It is called Greenwich civil (or Greenwich mean) time (G. C. T. or G. M. T.). The civil time day begins at midnight. Local civil time is designated L. C. T.

The *Equation of Time* is the amount (up to 16 minutes), varying from minute to minute of the day and through the year, which must be added to or subtracted from apparent time to give civil time and vice versa. The N. A. gives it for every 2 hours with plus or minus sign indicating procedure for converting civil to apparent time.

Transit of a body occurs the instant its point in the celestial sphere is on the meridian of the observer or on the meridian 180° away. When the transit is over the meridian which contains the zenith, it is designated as upper; when over the meridian 180° away, as lower.

Sidereal or *Star Time*. This is measured by the apparent daily motion of the Spring equinox or "First Point of Aries" (♈) around the earth. As this point is not marked by any heavenly body, we are dependent on the astronomers to calculate its position and publish in almanacs the right ascensions of bodies measured eastward from it.

"The equinox itself cannot be observed, being merely the intersection of two abstract lines upon the sky; but by a very long chain of observations, going back continuously to Hipparchus 150 B. C., and earlier, the relative spacing on the sky of all the lucid stars, and a great many more, has been determined with continually increasing accuracy, together with the 'proper motion' belonging to each. Relative to this mass of material, the position of the equator and ecliptic are assigned, or what comes to the same thing, the coordinates of each star are given relative to the equinox and equator." (*Encyclopedia Britannica,* 14th ed., Vol. 22, p. 227, "Time Measurement.")

A further difficulty comes from the constant slow westward shift or precession of the equinox. This is opposite to the movement of the earth in its orbit and amounts to 50″ of arc on the celestial sphere per year, and shortens the year by about 20 minutes. (For this we do not use the table given in Chapter 1 under Right Ascension which matches 24 hours with 360° because here we are figuring on a whole year for 360°.)*

The year, then, from one equinox and back to the same

* Now 360° = 1,296,000 seconds of arc. And 365 days = 525,600 minutes of time. So to show the time which the year loses by this 50″ shift:

$$50'' : 1{,}296{,}000'' :: X^m : 525{,}600^m$$
$$1{,}296{,}000\ X = 26{,}280{,}000$$
$$X = 20^m +.$$

is 20 minutes shorter than the year from a certain position relative to a "fixed" star and back to the same. These two kinds of years are known respectively as Solar or Tropical (equinox to same) and Sidereal (star to same).

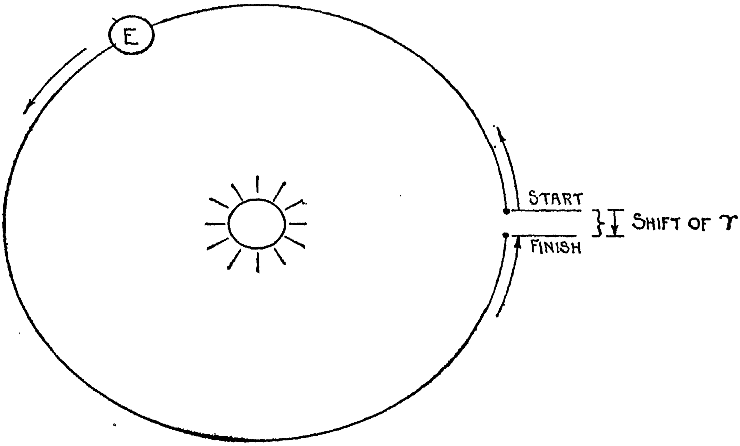


FIG. 7. The Solar Year.

The Solar Year equals an incomplete revolution of the Earth because of the westward shift of the Vernal Equinox (♈).

The solar year has: 365.2422 civil days, or 366.2422 sidereal days.

The sidereal year has: 365.25636 civil days, or 366.25636 sidereal days.

The solar year is the more satisfactory for calendar purposes since it keeps the seasons in a constant relation to dates. It is the one used in general and in the N. A.

The .2422 fractional day of the solar year (just under 1/4)

is what leads to our leap year system. To add a day to the calendar every 4 years gradually amounts to too much. So every year, whose number is divisible by 4, is a leap year, excepting the last year of each century (1900, for example) which is a leap year only when the number of the century is divisible by 4. This keeps the calendar in almost perfect order.

The matter of the two kinds of year is brought up to help prevent confusion which may arise from the fact that Bowditch (1938, p. 146) refers only to the sidereal year, and Dutton (7th ed. 1942, p. 248) only to the solar or tropical year.

Remember, then, that in navigation, whether using civil or sidereal time, it is always part of a solar year—not of a sidereal. (*See* FIG. 7.)

The sidereal day is almost exactly the time of one complete rotation of the earth on its axis.* These days are practically uniform. The sidereal day at any meridian begins with the transit of the "First Point of Aries" (♈) over that meridian. The date is usually not used but sidereal time is spoken of as of a certain date and hour of G. C. T.

A sidereal day is $23^h\ 56^m\ 4^s.1$ of civil time or $3^m\ 55^s.9$ (civil) shorter than a civil day.

* To find actual difference between the sidereal day and a complete rotation day:

Precession of ♈ in 1 year = 50″ of arc

Precession of ♈ in 1 sidereal day $= \frac{50''}{366} = .137''$ of arc

Now 15″ arc = 1 second time for earth's movement, so

$.137'' : 15'' :: X^s : 1^s$

$15\ X = .137$

X = .009 seconds time = the amount by which 1 sidereal day, between transits of ♈, is shorter than 1 complete rotation day.

A civil day is $24^h\ 3^m\ 56^s.6$ of sidereal time or $3^m\ 56^s.6$ (sidereal) longer than a sidereal day.

It will be seen that the difference between the shorter sidereal day and the longer civil day is $3^m\ 55^s.9$ in civil or $3^m\ 56^s.6$ in sidereal—the sidereal quantity being larger by .7 sec.

The actual lapse of time of this difference is the same, however expressed. But the sidereal units—days, hours, minutes, seconds—are all shorter than similar civil units, so that *for any time interval below the year, measurement in sidereal units will always show a larger quantity than measurement in civil units.*

Remember there are two senses in which both civil and sidereal quantities may be interpreted, as follows:

Civil—(a) Angular distance of mean sun from lower transit; and (b) A lapse of time since mean sun made lower transit.

Sidereal—(a) Angular distance of ♈ from upper transit; and (b) A lapse of time since ♈ made upper transit.

Now civil and sidereal time in the "a" sense are in similar units and can be combined without conversion.

But civil and sidereal time in the "b" sense are expressions of duration in different units and cannot be combined to find sum or difference without converting one into terms of the other. (N. A. supplies tables to use in converting either way.)

Of course an "a" sense of one kind cannot be combined with a "b" sense of the other.

Apparent Time is found in navigation by sextant observation of the sun and calculation therefrom of its hour

TABLE 4

TIME

Apparent or Solar	*Civil or Mean*	*Sidereal or Star*
True sun	Imaginary sun	First Point of Aries (♈)
In ecliptic	In equinoctial	In equinoctial
Day starts at lower transit	Day starts at lower transit	Day starts at upper transit
Days unequal: (24^h Civil ± Eq. T.)	Days equal: 24^h Civil = $24^h\ 3^m\ 56.6^s$ Sidereal	Days equal: 24^h Sidereal = $23^h\ 56^m\ 4.1^s$ Civil
Year = 365.2422 days	Year = 365.2422 days ("Solar")	Year = 366.2422 days ("Solar")
Obtained by:	Obtained by:	Obtained by:
* Sextant for H. A. of sun	Apparent ± Eq. T.	Sextant for H. A. of star
Civil ± Eq. T.	Sidereal converted	* Civil converted
Observed transit (on land)	* Chronometer corrected by:	Sidereal chronometer
	a. Its rate of change	Observed transit (on land)
	b. Radio time signals	

* Customary method in navigation.

angle (to be explained later) or by conversion of civil time by the equation of time.

Civil Time is not directly observable but is obtained by conversion of either apparent or sidereal. Practically it is taken from the ship's chronometer, corrected by the known rate of change of the chronometer or by radio time signals.

Sidereal Time is found by sextant observation of moon, planet or star, and calculation therefrom of the body's hour angle which is then combined with its R. A..obtained from the N. A. It may also be obtained by conversion of civil and sometimes is taken from a sidereal chronometer.

A rough estimate of local sidereal time (L. S. T.) may be made as follows: Imagine a line from the pole star drawn through β Cassiopeiae (Caph) and α Andromedae (Alpheratz) to the celestial equator. (*See* Chap. 25.) It will cut the equator approximately at the Vernal equinox. When this line is on the meridian through zenith the L. S. T. is 0^h. An estimate of its swing around the pole star counterclockwise may easily be made by dividing a circle around the pole star into quarters (6 hours each) and these quarters into thirds (2 hours each).

We must now consider the definitions of some terms that are in very frequent use.

A *great circle* of any sphere is one formed by the intersection on the sphere's surface of a plane which passes through the sphere's center. The earth's equator, and any meridian (with its opposite one) which passes through both poles, are examples. The shortest distance, on the surface, between any two points on a sphere is always part of a

great circle. The equinoctial is the great circle on the celestial sphere produced by intersection of the plane of the earth's equator. The ecliptic is another.

A *small circle* of a sphere is one described by the intersection on the sphere's surface of a plane which passes through the sphere but not through its center. All the parallels of latitude above and below the equator are small circles. Any circle on a sphere which includes less than half the sphere is a small circle.

An *hour circle* is a great circle on the celestial sphere passing through the poles and some point in question such as the projection of Greenwich or the projection of a heavenly body, as seen from the earth's center.

The *hour angle* of a body is the angle at the pole of the celestial sphere between the hour circle of the body and the celestial meridian of the observer. It is also measured by the arc of the celestial equator between the hour circle and the celestial meridian. It is either reckoned positively to the west all around to 360° or 24 hours, or, if over 180° west, is subtracted from 360° and designated east.

Greenwich hour angle is designated G. H. A. and local hour angle L. H. A. It is becoming customary to limit the use of L. H. A. to any westward measurement and to use *t* for an eastward or westward angle below 180°, which is then known as a *meridian angle*.

Table 4 may now be studied to review the three kinds of time.

Diagrams are most useful in visualizing problems in time or in position finding. It is much easier to record apparent movements of heavenly bodies on the edge of a circle clock-

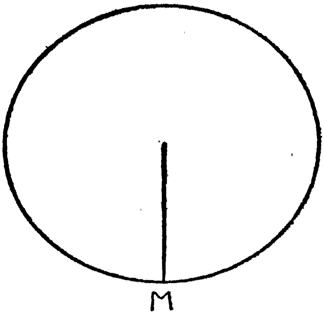


FIG. 8

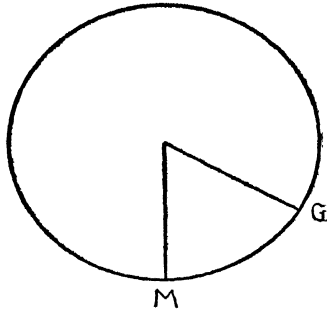


FIG. 9

Figure 8. A circle represents the celestial sphere in the plane of its equator and seen from above its north pole. A dot in the center represents earth. (We use a similar dot and circle to represent the earth and the plane of a given meridian of the C. S. when working latitude problems.) The circle, of course, stands for 360° or 24 hours. A line is drawn to the bottom part to indicate a projection of part of the meridian of the observer and always labeled M.

Figure 9. Another line is drawn as a projection of the meridian of Greenwich. It is placed approximately according to the supposed longitude and labeled G. Here it shows observer at about longitude 60° W.

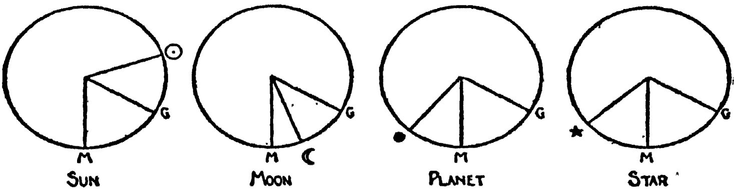


FIG. 10

Figure 10. Other lines represent hour circles of heavenly bodies as indicated with symbols. One of them is filled in after an observation has been made and the local hour angle has been calculated, as will be explained later. We may here recall once more that M, G, and the central dot are really in counterclockwise motion while any line for a heavenly body is in apparent clockwise motion.

wise, with stationary earth in center, than to record the actual rotation of the earth at the center of the circle counterclockwise. A simple convention has therefore been worked out. It may be done free-hand and only approximately but will show relationships clearly if we remember that the clockwise movement is only apparent.

Examples of the chief uses of such diagrams will be seen in Figures 8 to 17.

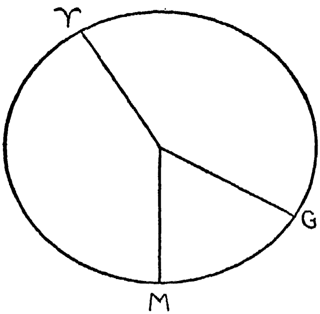


FIG. 11

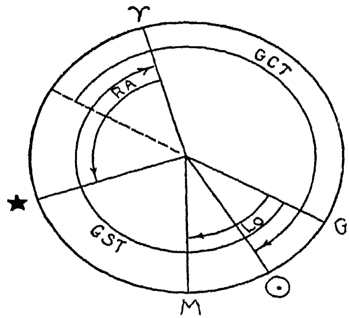


FIG. 12

Figure 11. The "First Point of Aries" (♈) or starting point of sidereal time may be similarly drawn in problems involving moon, planet or star. The Greenwich sidereal time would have to be first calculated. The N. A. gives it for 0^h G. C. T. of any date. Adding to this the actual G. C. T., and also a small amount from a table representing excess of sidereal over civil during that civil interval, gives G. S. T.

Figure 12 shows G. C. T., G. S. T., and R. A. of a star. Dotted line represents opposite meridian needed for start of G. C. T. Curved lines with arrows show amounts approximately as follows:

G. C. T.	14^h (2 P. M.)
G. S. T.	15^h
R. A.	6^h
Long. W.	60° (4 h.)

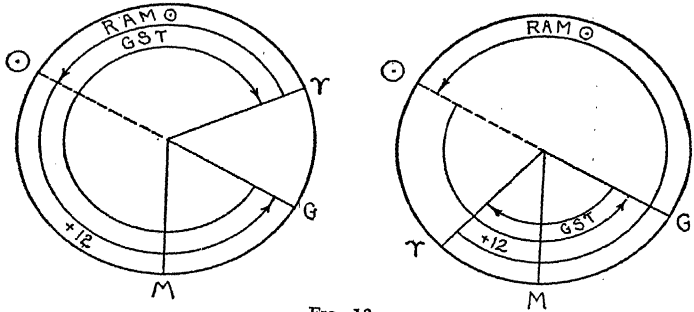
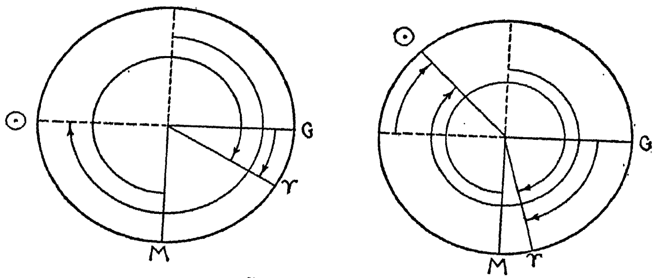


FIG. 13

Figure 13. Sidereal Time of 0^h G. C. T. is the same as R. A. of the Mean Sun + 12^h (—24 if over 24) at 0^h G. C. T. This latter expression is always abbreviated to R. A. M. ⊙ + 12. The two cases of G. S. T. given here will show this identity. The one on the right being over 24 h. requires a subtraction of 24.



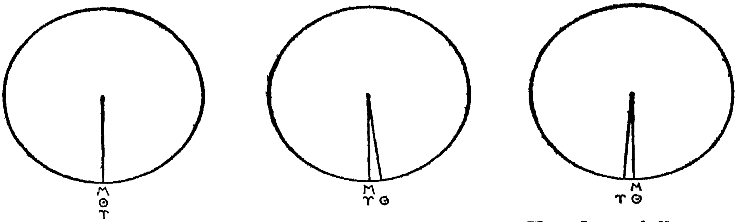
Long. = 90° W.

G.C.T.	0^h
G.S.T.	2^h
L.C.T.	18^h (day before)
L.S.T.	20^h (day before)

G.C.T.	3^h
G.S.T.	5^h + corr. for 3^h
L.C.T.	21^h (day before)
L.S.T.	23^h + corr. for 3^h

FIG. 14

Figure 14 shows two diagrams and the data they represent. The student may care to label the curved arrows with the proper designations.



Monday noon with	Tuesday Sidereal noon	Tuesday civil noon
L.S.T. 0	L.S.T. 0	L.S.T. $0^h\ 3^m\ 56^s .6$
L.C.T. 12	L.C.T. $11^h\ 56^m\ 4^s .1$	L.C.T. 12

FIG. 15

Figure 15, not to scale, is to illustrate the time difference between sidereal and civil days.

The local hour angle (L. H. A. or *t*) is the starting point of the actual navigational calculations for position. (*See* FIG. 16.) Whatever system is used from *t* on, the calculation of *t* must first be done. Table 5 shows how the N. A. method giving G. H. A. in arc has shortened it.

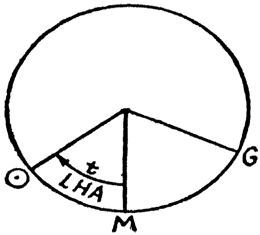


FIG. 16

TABLE 5
CALCULATIONS OF t

OLD WAY FOR SUN	OLD WAY FOR OTHERS	NEW WAY FOR ALL
G. C. T.	G. C. T.	G. C. T.
Eq. T.	R. A. M. ⊙ + 12	G. H. A. (arc)
G. A. T.	Corr.	Long. D. R.
G. H. A. (time)	G. S. T.	L. H. A. or *t*
G. H. A. (arc)	R. A.	
Long. D. R.	G. H. A. (time)	
L. H. A. or *t*	G. H. A. (arc)	
	Long. D. R.	
	L. H. A. or *t*	

The arithmetic from W* through G. C. T. to t probably offers more inducements for errors than the actual calculation which follows t. The student is cautioned to give this preliminary portion of all problems his most careful attention.

Apparent time may be described as the hour angle of the true sun westward from the observer's meridian +12 (−24 if over 24). This statement may sound a bit obscure but its truth can be realized by an examination of the two cases shown in Figure 17. The one on the left is obvious. The one on the right requires a subtraction of 24 which leaves the arc MX. This is of course equal to the arc $Y\odot$ which is the required time.

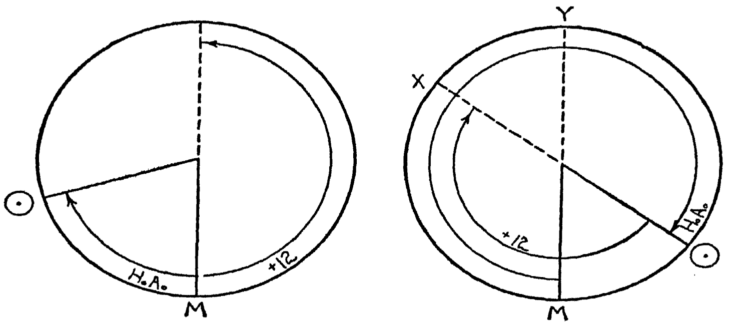


FIG. 17

Civil time is found by a similar rule for the mean sun.

Local apparent time is ordinarily found as follows: If body is east of observer's meridian, subtract local hour angle

* Watch time.

(in time) from 12. If body is west of observer's meridian, add 12 to local hour angle (in time).

Today practical navigation can ignore sidereal time. The N. A. gives data in terms of G. H. A. in arc sufficient for all ordinary purposes and even for star identification. This was only begun in the 1934 issue and is not preferred by all as yet. But to try to grasp the scheme of things in *Nautical Astronomy* without understanding something of sidereal time, is unwise.

For example, the new *American Air Almanac* makes use of two expressions of sidereal origin: G. H. A. ♈, which, of course, is the same as G. S. T., and S. H. A., which means Sidereal Hour Angle and is the body's angle westward from ♈. This is the same as 24 h. or 360° − R. A. of the body. The sum of G. H. A. ♈ + a correction + S. H. A. * (−24 h. or 360° if over 24 h. or 360°) = G. H. A. *.

Remember the following relationships:

G.H.A.	combined	with	L.H.A.	gives	longitude	in	degrees
G.A.T.	"	"	L.A.T.	"	"	"	time
G.C.T.	"	"	L.C.T.	"	"	"	"
G.S.T.	"	"	L.S.T.	"	"	"	"
G.H.A.			longitude in degrees gives L.H.A.				
G.A.T.			" " time gives L.A.T.				
G.C.T.			" " " " L.C.T.				
G.S.T.			" " " " L.S.T.				

G.H.A. (in time) combined with R. A. gives G.S.T.
G.S.T. combined with R.A. gives G.H.A. (in time)
L.H.A. (in time) combined with R. A. gives L.S.T.
L.S.T. combined with R.A. gives L.H.A. (in time)

Standard or *Zone Time* is based on the L. C. T. of meridians at 15° intervals from Greenwich. For convenience of railways and everyday affairs, a standard time zone extends 7.5° each side of these meridians in which the time of the meridian is used. This system has been extended over the oceans and is used by the navies of the United States, Great Britain, France and Italy. The Greenwich zone is called Zero Zone. Each other zone is numbered from 1 to 12 according to the hourly difference from Greenwich. East zones are called minus zones since in each of them the zone number must be subtracted from the standard time to obtain the G. C. T. Conversely, west zones are called plus. The twelfth zone is divided medially by the 180th meridian and the terms "minus" and "plus" are used in the halves of this zone which lie in east longitude and west longitude, respectively. (*See* Table 6.) These zone boundaries are modified in the vicinity of land for special conditions and circumstances. Instead of adjusting the ship's time to apparent time at noon each day, the clock is adjusted to the standard time of the successive zones as they are entered, the change invariably being exactly one hour. When it is desired to obtain zone time from G. C. T., the sign of the zone in question must be reversed and the result applied to the G. C. T. Zone time has simplified the work of the navigator in many ways. His watch is usually kept on it.

Certain relations between Z. T. and L. C. T. may confuse one at first. A study of Figures 18 and 19 should clear up this problem.

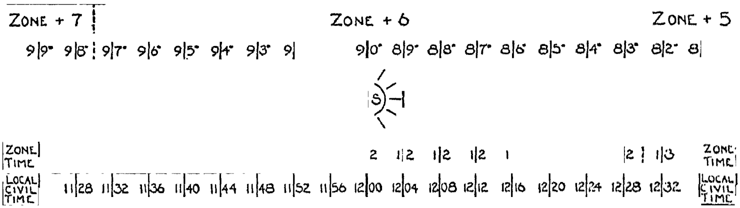


FIG. 18. Relations between zone time and local civil time. The figure represents one instant only.

Mean sun is over 90th meridian W. Longitude.
Zone + 6 time is 12 noon.
All other meridians in Zone + 6 have zone time 12.
Zone + 5 is one hour more.
Zone + 7 is one hour less.
L.C.T. of 90th meridian is 12 noon.
L.C.T. of meridians to east increases 4 minutes for each. (Fast of Z.T.)
L.C.T. of meridians to west decreases 4 minutes for each. (Slow of Z.T.)

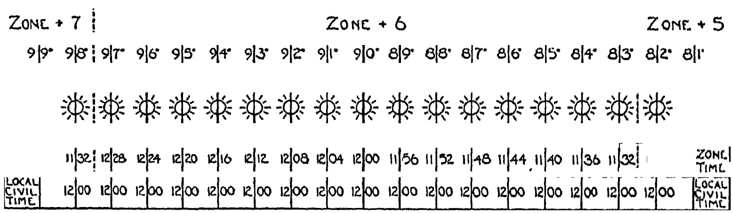


FIG. 19. Zone time of mean sun noons in zone.

The figure represents 17 different instants. It shows at what zone time the mean sun will be over each meridian.

TABLE 6

ZONE TIME

Zero Zone: Long. 7½ W.–7½ E.

W. LONG Zones	LIMITS	E. LONG Zones
+ 1	7½– 22½	– 1
+ 2	22½– 37½	– 2
+ 3	37½– 52½	– 3
+ 4	52½– 67½	– 4
+ 5	67½– 82½	– 5
+ 6	82½– 97½	– 6
+ 7	97½–112½	– 7
+ 8	112½–127½	– 8
+ 9	127½–142½	– 9
+10	142½–157½	–10
+11	157½–172½	–11
+12	172½–180	–12

It is possible through daily radio signals to keep a second-setting watch correct for G. C. T. However, many ships still have no radio and use only the standard chronometer which gains or loses at a certain rate. Most of the textbook problems in navigation start the data with the observer's watch time and proceed as follows:

W	(Watch)
+*C*–*W*	(Chronometer minus Watch; add 12 h. to C. if necessary)
C F	(Chronometer Face)
±*C C*	(Chronometer Correction)
G C T	(Greenwich Civil Time)

A newer form of recording the data for G. C. T. is as follows:

W.	(Watch)
±*W. E.*	(Watch Error)
Z. T.	(Zone Time)
Z. D.	(Zone Description)
G. C. T.	(Greenwich Civil Time)

Sights had best be taken with a stop-watch unless an assistant is available to note time when observer calls for it. Taking a sight with a stop-watch in the left hand is simple and the stem may be punched at the instant desired. The stop-watch can then be taken to the chronometer and stopped when the latter reads some even minute. Subtracting the stop-watch total gives the chronometer reading at time of sight. This can be abbreviated as follows:

C. S.	(Chronometer at stop)
−*R. W.*	(Ran stop-watch)
C. O.	(Chronometer at observation)
±*C. C.*	(Chronometer correction)
G. C. T.	(Greenwich Civil Time)

Greenwich Date

Because chronometers do not have 24-hour dials, there is always a problem, in applying a chronometer correction, whether to add 12 hours for a P. M. hour at Greenwich. Also the G. date may be that of the ship, one earlier, or one later. Dutton gives the following quick and easy mental method of G. C. T. and date determination:

1. Apply the zone description to the ship's approximate zone time, obtaining approximate G. C. T.
2. If it is necessary to add 24 hours to ship's time in order to subtract a minus zone description, the G. date is one less than ship's.
3. If after applying the zone description the total is over 24 hours, the excess is the G. C. T. and the G. date is one more than ship's.
4. Otherwise the G. date is same as ship's.

One can diagram the problem by making a circle for equinoctial and a dot in the center for north pole. West is clockwise. Then draw projected local meridian to bottom and G. meridian according to longitude. Draw dotted lines for their opposite (or lower branch) meridians. Draw symbol for sun on circumference according to L. C. T. or Z. T. and connect it to center.

G. C. T. will at once appear less or more than 12. If the latter, add 12 to chronometer.

G. date is the same as local unless sun lies in the sector between lower branches.

G. date is one more than local if sun is between lower branches and west of G. lower branch.

G. date is one less than local if sun is between lower branches and east of G. lower branch.

Change of Date

Greenwich Civil Noon is a unique instant. It is then, and only then, that the same date prevails all over the earth.

We might discuss this matter of date by referring to the sun's apparent motion around the earth, but it is felt that

a sounder conception will be gained by sticking to the real situation and referring to the earth's rotation opposite a stationary sun. For convenience, we can disregard the progress of earth in its orbit and the fact that a little more than one rotation occurs between sun noons.

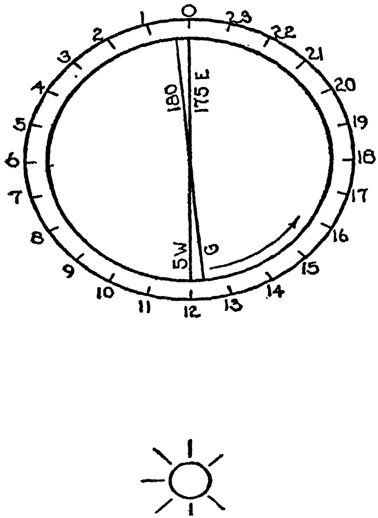


FIG. 20. Time Frame.
Earth seen from above North Pole rotating East. Time frame outside does not change position relative to Sun. G. C. T. = $12^h\ 20^m$.

Take it on faith, for the moment, that at Greenwich Civil Noon of July 4 this date prevails throughout the earth.

The instant the Greenwich meridian has passed eastward under the mean sun, the 180° meridian (also known as the International Date line) will have come around closer to the mean sun by an equal amount.

Say these amounts are each 5° or ⅓ of an hour. G. C. T. will be 4 July $12^h\ 20^m$ and L. C. T. at 180° will be 5 July $0^h\ 20^m$, that is, 20 minutes past midnight with a new date. (*See* FIG. 20.) Similarly:

L. C. T. at Long	179° E.	will be 5 July	$0^h\ 16^m$.
	178°	"	$0^h\ 12^m$.
	177°	"	$0^h\ 8^m$.
	176°	"	$0^h\ 4^m$.
	175°	"	$0^h\ 0^m$.

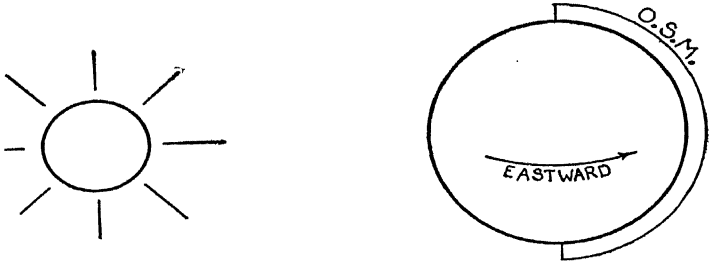


FIG. 21. "The Opposite-the-Sun" Meridian.

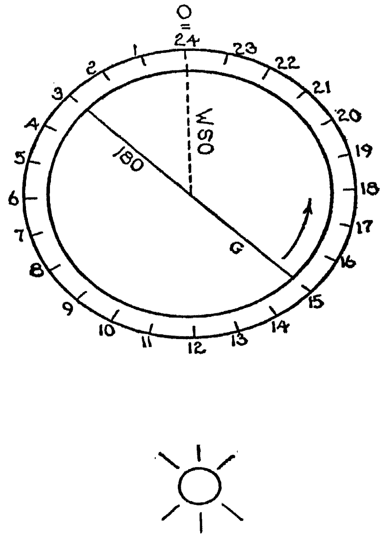


FIG. 22. Time Frame and O. S. M.

Time Frame and O. S. M. each retains its position relative to Sun while earth rotates east. Above = G. C. T. 15^h.

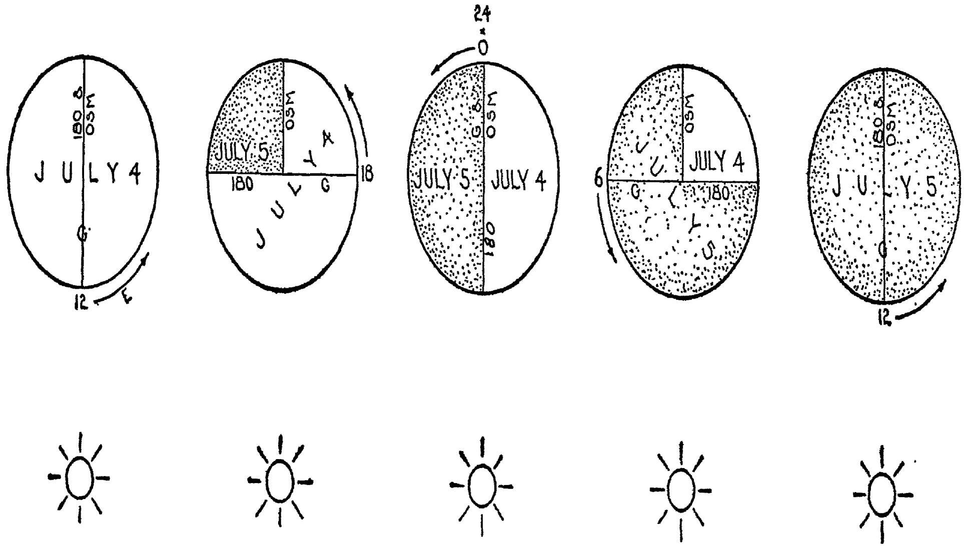


FIG. 23. Change of Date.

When 180th passes under O. S. M. a new date is "*Born.*"
O. S. M. "*Sprays*" the Rotating Earth with this new date until 180th again passes under O. S. M. and another date is born.

The 175th E. meridian is now opposite the sun as was the 180th at Greenwich noon. Naturally, any earth meridian opposite the sun is experiencing midnight or the start of a new day.

Let us think now of a permanently "opposite-the-sun Meridian" (O. S. M.) not rotating with earth, but like a half hoop suspended at some distance above the earth's surface by attachment of its ends to the "poles." The earth rushes eastward under this O. S. M. (*See* FIGS. 21-22.) All the earth's surface from this O. S. M. eastward to the 180th meridian has the new date. As the 180th proceeds eastward with the earth's rotation, more and more area of earth's surface is brought between it and O. S. M. By the time the 180th comes under the sun at noon, Greenwich has reached its midnight and changes date to July 5. The half of earth east from Greenwich to the 180th is likewise July 5, while the opposite half is still July 4. As the 180th completes the remaining half rotation reaching midnight again and Greenwich reaches noon of July 5, the entire earth again has one date, July 5. Figure 23 will make this clear.

Crossing the 180th Meridian

A glance at Figure 23 will show that a ship sailing westward across the 180th must add one to the date, while one crossing eastward must subtract one from the date. In each instance, the name of the longitude (E. or W.) must be changed.

The student may wonder about a ship passing the O. S. M. This, of course, is not an earth mark, but one kept opposite the sun. A ship on earth is carried east under

O. S. M. by the earth itself at the speed of its rotation, and passes midnight in consequence. No ship could travel fast enough (except near the poles) to pass under O. S. M. westwardly as that would require a speed greater than the speed of earth's rotation.

For practical convenience, separating continents, keeping certain groups of islands under one time, etc., the International Date Line is not just the same as the 180th meridian. A glance at a globe will show the several angles and curves that have been agreed upon.

Chronometers

Before the invention of a timekeeper which would remain close to correct in varying temperatures, navigators were never certain of their longitude. About 30 miles was as close as they could come by old methods of computation, chief of which was through the measurement of "lunar distances." This meant getting the angle between the edge of the moon and some other body by sextant, and noting the time, from which, by elaborate corrections and computations, the moon's right ascension could be found. As this changes about 30″ of arc in every minute of time, it was necessary to observe within 30″ of the correct distance to be correct within 1 minute. Consulting the almanac showed at just what G. C. T. the moon had this R. A. Comparing this G. C. T. with the ship's clock at observation showed the error of the latter. Having thus obtained a fairly correct G. C. T., observations could later be made of the sun to get hour angle and hence Local Apparent Time. G. C. T. with the equation of time gave

G. A. T. Then L. A. T. compared with G. A. T. gave Longitude.

John Harrison (1693-1776) was an English watchmaker. When he was twenty years old, the British Government offered a prize of 20,000 pounds for a method which would determine longitude within 30 miles. Fifteen years later, Harrison invented a compensating grid-iron pendulum which would maintain its length at all temperatures, and applied unsuccessfully for the prize. By 1761, Harrison had produced a better instrument. In 1764, this was taken to Jamaica and back on a voyage of over five months and showed an error of only 1^m 54.05^s. It depended on the unequal expansion of two metals with change of temperature. The British Government awarded the prize to Harrison who, however, did not receive it until nine years had passed. Two years later, in 1776, he died at the age of 83.

The usual ship's chronometer of today is a finely made instrument kept in a box and swinging on gimbals to keep it level, which is wound regularly but not set after it is once started correctly. The rate of change is noted and correct time is calculated by applying this rate. It is customary for a ship to carry three so that a serious error in one will be manifest by its disagreement with the other two. Chronometers are made with 12-hour dials which sometimes necessitates adding 12 hours to the face to get G. C. T.

Recently there have appeared what are called "second-setting" watches. One model designed by Commander Weems and in wrist-watch size, makes second hand setting possible, without stopping the movement, by a device which rotates the dial under the second hand. Another

model, which has the advantage of a 24-hour dial, has a lever which can stop the movement until the operator pushes the lever back. On a small boat where radio time signals can be had daily, this watch is probably an adequate substitute for a chronometer.

3. The Nautical Almanac

THE AMERICAN NAUTICAL ALMANAC is issued annually by the United States Naval Observatory and may be obtained for the current or coming year for 65 cents in money order, from the United States Government Printing Office, Washington, D. C.

The N. A., as it is called, is one of the four absolute essentials of equipment for doing celestial navigation. The other three are the sextant, the chronometer, and one of the many types of tables necessary for computation.

The student should spend sufficient time in looking through the N. A. and reading the explanations printed in its last pages to become thoroughly familiar with it.

As has been previously stated, navigational computation has been shortened and simplified by the recent inclusion of Greenwich Hour Angle for all bodies used. This and the declination are the principal items for which we use the N. A. The bodies, for which data are given are the sun, moon, Venus, Mars, Jupiter, Saturn, and 54 convenient and prominent stars. Special tables are provided for computing latitude from Polaris and several other useful reference tables are given.

Data are given at the following intervals:

	Dec.	*G. H. A.*
Sun	2 h	2 h
Moon	1 h	1 h
Planets	1 day	1 day
Stars	1 month	1 day

Following the data for each of the bodies except the sun, there will be found tables for computing any values of declination and G. H. A. which lie between the values given in the main tables. These save a great deal of arithmetical interpolation. The sun's declination is easily interpolated by inspection and its G. H. A. is corrected by a table given on every third page. Star declinations hardly vary from one month to the next.

While the use of sidereal time will not be advocated in this book, many professional navigators use it and so the tables, except those for the sun, give right ascension for all the bodies. The first table in the N. A. gives sidereal time of 0^h civil time at Greenwich for every day in the year, and another table (No. VI in 1943) gives the time which must be added to the above with the excess of G. C. T. over 0^h in order to give Greenwich Sidereal Time.

The equation of time is given in the sun tables for conversion of civil to apparent time. We shall find little use for it.

A two-page table of mean places of 110 additional stars is provided. It will be useful occasionally when a single star is observed and proves to be neither Polaris nor a planet nor one of the usual 54. In such case it may be among these 110 and the older method of sidereal time will have to be used to obtain *t* (the L. H. A.) since the R. A. values are given without the G. H. A. data.

Tables are included for calculating times of sunrise, sunset, moonrise, moonset, and twilight.

Commander Angas in *An Introduction to Navigation* (Vol. XIII. of Motor Boating's Ideal Series, p. 40) suggests entering abbreviations for the month in the upper part

of the star chart given in the N. A. as follows: At upper right corner put Nov. Then passing to the left and skipping one square, put Dec. Continue with Jan., etc., in every *other* square to the *left* until Oct. occupies the second square from upper left corner. "At nine p. m. local time in the middle of each month, the observer's meridian will about coincide with an imaginary vertical line on the chart drawn through the center of the space occupied by the month in question."

The writer has found it a great convenience to make up a correction table booklet from several old almanacs. Bored with having to hunt through the almanac for the proper correction tables following the given body's data, he has cut out the necessary tables (which do not change from year to year), pasted them on loose-leaf sheets of regular typewriter size paper using one side only and bound them in a 10 cent binder. Index markers make it easy to turn at once to all the correction tables needed for any body observed. Certain tables appear more than once, but this makes each division complete in itself. Altitude correction tables (to be discussed later) are included and, since these appear in still another part of the almanac, further saving of time is made possible. The arrangement is as follows:

SUN

Pages

1. Height of Eye (Table C).
   Altitude (Table A, sun portion).
   Additional altitude for semidiameter (Table B).
2. Greenwich Hour Angle.

MOON

3. Height of Eye (Table C).
4–5. Altitude (Table D).
6–7. Greenwich Hour Angle.
8. Declination.

PLANET

9. Height of Eye (Table C).
Altitude (Table A, star portion).
10–13. Greenwich Hour Angle.
14–17. Declination.

STAR

18. Height of Eye (Table C).
Altitude (Table A, star portion).
19–21. Greenwich Hour Angle.

OTHER TABLES

22–23. Sidereal into Mean Solar (Table V).
24–25. Mean Solar into Sidereal (Table VI).
26–28. Proportional Parts (Table VII).
29. Arc to Time (Table VIII).
30. Star transit corrections.

The *American Air Almanac,* published once in 1933 and then discontinued until 1941, is a simplified almanac issued in sections covering 4 months each. It is arranged so as to usually give the desired data from a single opening. All values are to the nearest minute of arc. Interpolation is unnecessary. Dutton says it "can be used for surface navigation in open waters without fear of introducing any serious error. Near land it should only be used with caution because errors resulting from its use are not the only errors to be expected in the observed position."

4. Altitudes

THE CELESTIAL HORIZON of an observer is the great circle of the celestial sphere that is everywhere 90° from his zenith. At the extreme distance of the celestial sphere it makes no difference whether the observer is on the surface of the earth or is theoretically at the center of the earth—the great circles in these cases will practically coincide. Altitudes of heavenly bodies are expressed in angular distance from the celestial horizon with the observer imagined to be at the earth's center. For the far distant stars, there will be no difference if the altitude is measured from the earth's surface, but for nearer bodies —sun and moon—there must be a correction to our actual observation.

True Altitude of a heavenly body, therefore, at any place on the earth's surface, is the altitude of its center, as it would be measured by an observer at the center of the earth, above the plane passed through the center of the earth perpendicular to the direction of the zenith.

Sextant Altitude, as measured at sea, must be converted to the true altitude by application of corrections for certain items as follows:

For sun and moon:

Index correction
Refraction
Dip
Parallax
Semidiameter

For planets and stars:

Index correction Dip
Refraction

Index Correction will be explained under the chapter on the sextant and is merely to correct a mechanical error of that instrument that may be present.

Dip of the horizon, is the depression of the visible sea horizon below the celestial horizon due to the elevation of the eye of the observer above the level of the sea. Sextant altitudes taken from the bridge of a steamer or even the deck of a small yacht are enough larger than true altitudes to require correction by N. A. Table C (Height of Eye). The correction is, of course, subtracted (*see* Bowditch, pp. 153-4, for the influence of unusual conditions of temperature and barometric pressure).

Refraction means the bending which rays of light undergo when passing obliquely from one medium into another of different density. If the latter be *more* dense, the ray will be bent toward a perpendicular to the line separating the two media. The earth's atmosphere increases in density down to the earth's surface. Hence the path of an obliquely incoming ray of light, by passing from a rarer medium to one of increasing density, becomes a curve concave toward the earth. The last direction of the ray is that of a tangent to the curved path at the eye of the observer. The difference of this from the original direction of the ray is the refraction. Refraction therefore increases the *apparent* altitude of a heavenly body. It does not change its direction laterally. At the zenith, refraction is zero. At horizon it is greatest. The

correction must always be subtracted. (*See* FIG. 24.) Table A for sun, planet or star, and Table D for moon, include correction for refraction.

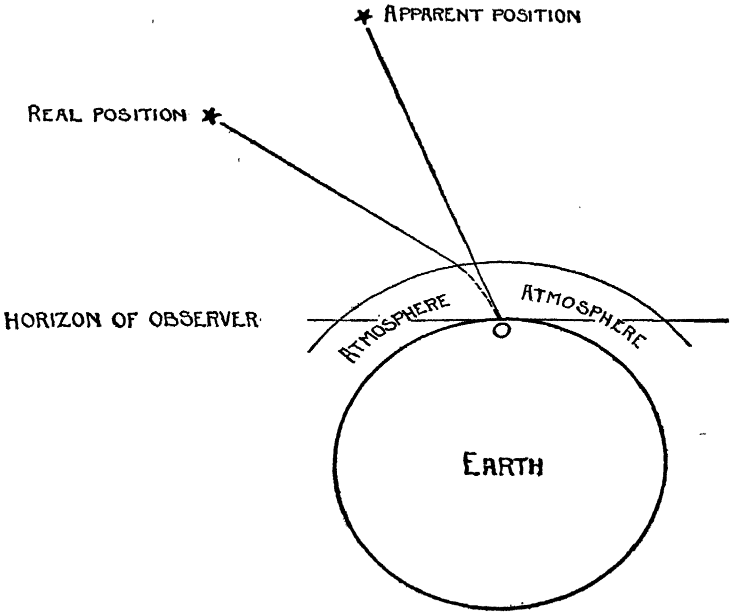


FIG. 24. Refraction.

Parallax of a heavenly body in general is the angle between two straight lines drawn to the body from different points. *Geocentric parallax*—the only kind with which we are concerned—is the angle subtended at a body by that radius of the earth which passes through the observer's

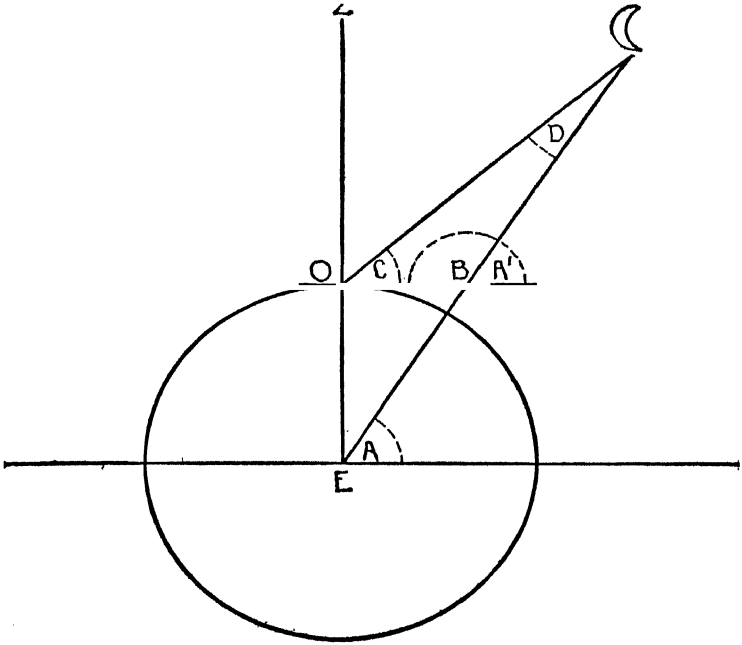


FIG. 25. Parallax.

☾ = Moon (Lower Limb) Observed.
D = Parallax (N. A.)
O = Observer.
E = Earth's Center.
C = Observed Altitude.
A = Altitude from Earth's Center = A' by Geometry.
$A' + B = 180°$, and
$C + D + B = 180°$, Therefore
$A' + B = C + D + B$, and
$A' = C + D$, Therefore
$A = C + D$.

Altitude from Earth's Center = Observed Altitude + Parallax.

position. *Horizontal parallax* is the maximum value of this parallax for a particular body and is present when the body is on the observer's horizon. *Parallax in altitude* is the parallax when the body is at any point above the observer's horizon. It diminishes to zero at the zenith. Putting it the other way around, parallax is the difference in altitude of a body supposedly measured at the same instant from a point on the earth's surface and, with parallel horizon, from the earth's center. This is shown in Figure 25. Parallax is always additive. Table A for sun and Table D for moon include corrections for parallax. The N. A. gives horizontal parallax for the moon on each page with the other moon data and we must note it for use with Table D to correct sextant altitude. Parallax of planets may be neglected in practical navigation.

Semidiameter of a heavenly body is half the angle subtended by the diameter of the visible disc at the eye of the observer. In cases of sun, moon and planets, whose distances from earth vary at different times, the semidiameters will change. The moon is nearer an observer when at zenith than when at horizon by the length of the earth's radius and the ratio of this length to the total distance of the moon is large enough to cause measurable enlargement. (This shows that our ordinary impression that the moon looks larger when low is an illusion. This effect is probably due to the nearness of earth landmarks, buildings and so forth, which make the moon seem large by comparison.) The increase in semidiameter due to increase in altitude is called *augmentation*. Semidiameter is to be added to observed altitude in case the altitude of the lower limb of a body has been measured and to be subtracted in the case

of the upper limb. Tables A and B include corrections for semidiameter for the sun and Table D for the moon. Semidiameter of planets may be neglected and of stars is not measurable in navigation.

For greater accuracy one should mentally make the subtraction for dip and the correction for index error first, and use the result in entering the table containing corrections for refraction, finally using the algebraic sum of the dip and other corrections to correct the sextant altitude.

5. The Sextant

THE FIRST ELEMENT required in any problem of celestial navigation is the angular altitude of the body observed. This altitude is measured by means of a sextant or an instrument of the sextant family. Measuring the angle is done by bringing into coincidence at the eye rays of light received directly from the horizon and by reflection from the celestial body, the measure being afforded by the inclination of a movable mirror to a fixed one. The handle, the triangular frame with apex above and scaled arc below, the telescope, eye-shade glasses, and horizon mirror, whose

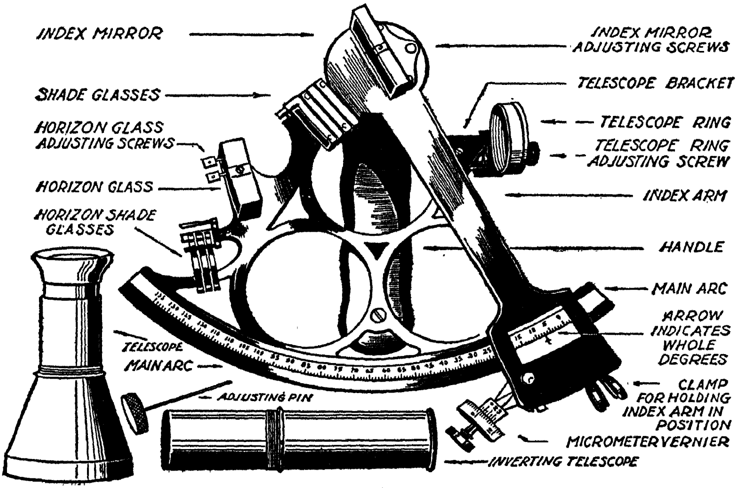


FIG. 26. Sextant.

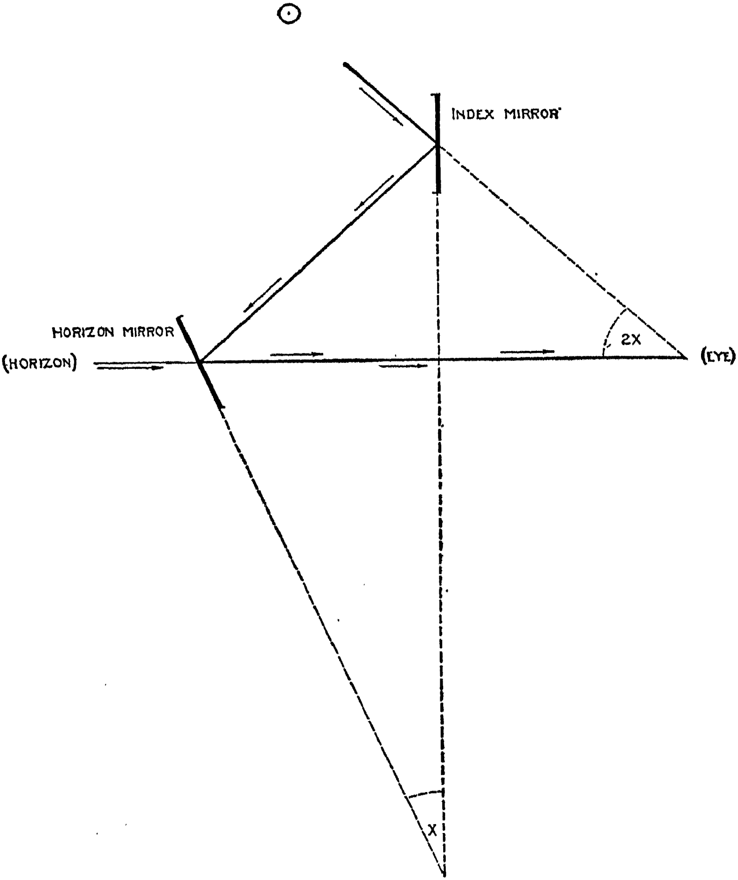


FIG. 27. Sextant Angles.

right half alone is silvered, are all rigid parts of a unit. The index mirror at top to catch the rays from the body is part of the movable arm which terminates below in the vernier scale with its screws and magnifying glass. This is moved along the arc, tipping the index mirror until the two images are brought together. It is then clamped by a screw on the right and the tangent screw on the left is used for finer adjustment. The scale is then read to give the angle of altitude.

In measuring the altitude of a celestial body, it is necessary that the angle shall be measured to that point of the horizon which lies vertically beneath the body. To determine this point the observer should swing the instrument slightly to the left and right of the vertical about the line of sight as an axis, taking care to keep the body in the middle of the field of view. The body will appear to describe an arc of a circle, convex down. The lowest point of this arc marks the true vertical.

When a ray of light is reflected from a plane surface, the angle of reflection is equal to the angle of incidence. From this it may be proved geometrically that, when a ray of light undergoes two reflections in the same plane, the angle between its first and its last direction is equal to twice the inclination of the reflecting surfaces. (*See* FIG. 27.)

The vernier is an attachment for facilitating the exact reading of the arc scale of the sextant by which certain fractional parts of the smallest division of the scale are measured. A sextant vernier is a shorter scale usually containing one more division than an equal length of the arc scale. Both arc scale and vernier readings increase to the left. To read any sextant it is necessary to observe the arc

The numbers are for each 10°.
The next highest marks are for 1°.
The next highest marks are for ½° = 30′.
The lowest marks are for 1/6° = 10′.

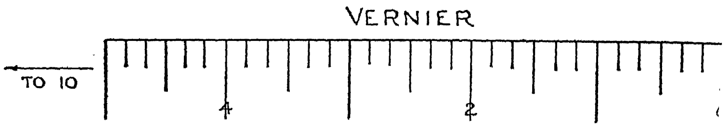


FIG. 28. Arc and Vernier Scales.

When 0 of Vernier is at 0° arc, 10 of vernier is at 19° 50′ of arc. This is a proportion of 60 to 119. The vernier has double spaces for clearer reading. Otherwise, the proportion would be 120 to 119.

scale division next to the right from the vernier zero and add thereto the angle corresponding to that division of the vernier to the left which is most nearly in exact coincidence with a division of the arc scale. Figure 28 shows the arc scale and vernier on a typical sextant.

Rule: The smallest measure to which a vernier reads equals:

$$\frac{\text{length of 1 division of scale}}{\text{number of divisions of vernier}} \text{ as } \frac{10'}{60} = 10'' \text{ or } \frac{10'}{120} = 5'' \text{ etc.}$$

Hence, after observation read thus. (*See* FIG. 29):

Scale: right of vernier zero to degrees and 10′ units.

Find line on vernier to left of its zero closest to a scale line.

Vernier number gives extra 1′ units.

Vernier marks give extra 10″ units.

Index Error exists when, with index and horizon mirrors parallel, the zero of the vernier does not coincide with the zero of scale. Observe a star, or the sea horizon in daylight, directly through telescope and move index until reflected image coincides with direct. If now the vernier zero is to left of the scale zero, all readings will be too great by the amount of this divergence; if to right of scale zero, readings will be similarly too small. (*See* Chap. 24 for details.)

Index Correction (I. C.) is expressed as + or − the amount of arc to be applied to the observed amount.

Certain minute errors due to construction and not correctable by adjustment are usually noted in certificates accompanying the instrument when purchased.

Certain adjustments must occasionally be made. *See* Dutton, pp. 219-20, for detailed information as to:

Index mirror	Telescope	Horizon mirror

Properly speaking, instruments of the sextant family should be designated as follows:

Octant:	45° arc measures angles to	90°
Sextant:	60° arc measures angles to	120°
Quintant:	72° arc measures angles to	144°
Quadrant:	90° arc measures angles to	180°

The author's instrument, therefore, with an arc of 75° for angles to 150° would seem to qualify as a Super-Quintant!

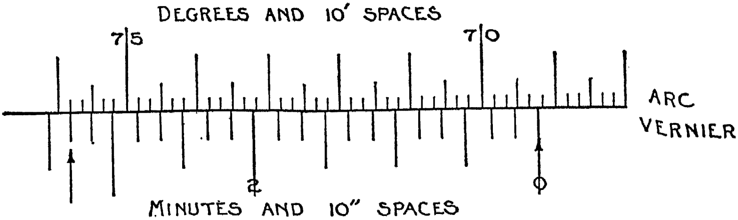


FIG. 29. Reading the Sextant.
Passing to left shows first coincidence at 75° 50′ of arc (Arrow).

Reading Arc	= 69° 10′
Reading Vernier	= 3′ 20″
Reading Total	= 69° 13′ 20″

The U. S. Navy now uses a sextant fitted with an endless tangent screw which carries a micrometer drum from which the minutes and the tenths of a minute of arc are read. The tangent screw is thrown out of gear by pressing a lever so the index arm may be freely moved. Releasing the lever stops the arm at individual degrees and throws the tangent screw into gear again for finer adjustment. (*See* FIG. 26.)

An artificial horizon may be provided by a glass-roofed dish of mercury or even a saucer of ink placed in an uncovered box with sides low enough to let the body's rays in but high enough to keep moving air from rippling the ink. The angle between the celestial body and its reflection in the artificial horizon is measured by sextant. Half of

this angle equals the altitude of the body. (*See* Chap. 24 for details.)

A *bubble sextant* is an instrument containing a mechanism which supplies an artificial horizon. It is much more expensive and less accurate than the standard type of sextant but can be used when the natural horizon is invisible, as in hazy weather, or in polar regions, on the desert, or in an airplane high above the earth and clouds.

Historical

The evolution of the modern sextant probably began with the astrolabe, used by early Greek and Arab astronomers. It consisted of a graduated circle suspended in the vertical plane from a ring at the top. At its center a sighting bar was attached somewhat as a compass needle and, by looking along this at a body and noting the scale on the circle, altitude could be measured. Elaborate forms of the astrolabe were in use in the sixteenth century.

Capt. John Davis, an English navigator, developed a quadrant in 1594 which had two arcs and required sighting in two different directions. A later form was used with the observer's back to the sun. Many subsequent instruments depended on a plumb line.

In 1729 Pierre Bouguer invented an instrument which needed only a sight of the horizon while a beam from the sun was kept visible in line on a wooden peg.

The cross-staff, something like a T square, was also in use at about this time and necessitated sighting sun and horizon separately. It too was used both facing and with back to the sun.

The double reflecting mirror instrument was suggested in 1674 both by Robert Hooke, a professor of geometry in London, and Sir Isaac Newton, independently, but no models seem to have been made.

In 1730 Thomas Godfrey of Philadelphia and John Hadley, an English astronomer, independently constructed double reflecting instruments much like our sextants of today. Hadley, who was Vice-President of the Royal Society of London, probably suppressed Newton's notes and certainly ignored Godfrey's claim and obtained the credit for the invention.

Capt. Campbell in 1757 enlarged the arc to make it a true sextant. Up to 1775 the instruments were of all-wood construction. Various improvements followed and the all-metal sextant appeared in the early nineteenth century. Verniers then came into use although described by Pierre Vernier long before, in 1631. Commander Hull of the U. S. S. *Constitution* in 1812 used a sextant which had shade glasses, telescope, vernier, ivory arc, brass fittings and ebony frame while, at this same period, a similar instrument constructed of brass with a silver arc was in use by Nathaniel Bowditch. Relatively minor improvements followed but the sextant of today is not radically different from its ancestor of 100 years ago. (See *The Evolution of the Sextant* by Commodore E. S. Clark, U. S. N. I. P. Nov. 1936, *Navigational Antecedents* by Commander H. D. McGuire, U. S. N. I. P. May 1933 and *The American Inventor of the Reflecting Quadrant,* also by McGuire, U. S. N. I. P. Aug. 1940.)

6. The Compass

KNOWLEDGE OF THE LODESTONE and its influence on a piece of iron touched by it is of great antiquity. Its use in a form to indicate directions at sea was a subsequent development. The Chinese, Arabs, Greeks, Etruscans, Finns and Italians have all been credited as originators of the compass. *Encyclopedia Britannica* (14th ed., Vol. 6, p. 176, "Compass") says . . . "the earliest definite mention as yet known of the use of the mariner's compass in the middle ages occurs in a treatise entitled *De utensilibus,* written by Alexander Neckam in the 12th century. He speaks there of a needle carried on board ship which, being placed on a pivot, and allowed to take its own position of repose, shows mariners their course when the polar star is hidden."

The Magnetic Compass of today as used on ships is the Thomson (Lord Kelvin) instrument introduced in 1876 with a few improvements. Kelvin used several magnetic needles in parallel attached under a circular card on which were printed the "points," the whole supported on a pivot for easy rotation. A subsequent improvement was filling the compass bowl with an alcohol mixture and sealing it under the glass cover, a corrugated chamber being provided for expansion of the liquid with increased temperatures. Instead of the flat glass top, a modern development is the "spherical" (really hemispherical) glass cover. Compasses are swung on gimbals to keep them level when the ship rolls or pitches, and mounted in a pedestal called a

binnacle. Through the bowl there is painted a thin black line from front to back. This is known as the lubber's line and the compass must be installed with this line exactly parallel to the ship's keel. The forward tip of the line, visible over the compass card, is the mark by which the helmsman notes the course, on the card, of the ship's head.

Compass cards are marked in two principal ways:

The old point system consists of 32 points around the circle, starting from north as follows:

North	Northeast by East
North by East	East Northeast
North Northeast	East by North
Northeast by North	East
Northeast	etc.

Each point is $11\frac{1}{4}$ degrees from the next. A system of quarter points is added, each being equal to about 2.8 degrees. The older (Merchant Marine) custom and the newer (Navy) custom of naming these quarter points are both somewhat difficult to memorize and of no real use to the student.

The other method of marking is in 360° around the circle clockwise. This system has many advantages aside from the fact that steamers can be steered to 1° while the smallest unit of the point system is about 3°. East is 90°, South is 180°, West is 270° and North is 360° or 0°. This system is all one needs at sea for celestial navigation.

Compass error results from two main causes, Variation and Deviation, now to be discussed.

Variation. The earth may be thought of as a great

magnet whose poles, however, do not exactly correspond to the geographical poles. The north magnetic pole is at about Lat. 70° N., Long. 97° W. and the south magnetic pole at Lat. 73° S., Long. 155° E. (As unlike poles attract and likes repel, the "north" end of the compass needle is really "north-seeking" or south.) As the north magnetic pole is above Hudson Bay and about 1200 miles below the geographical north pole, it will be evident that, in the

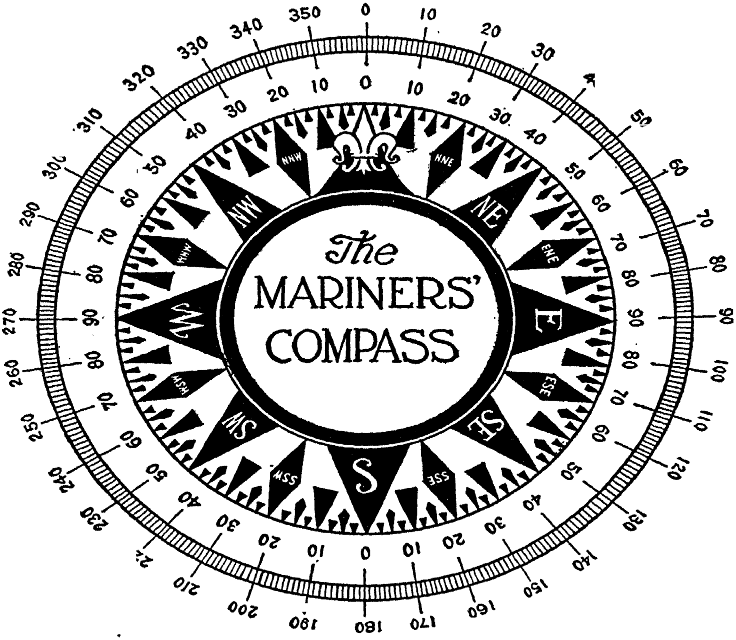


FIG. 30. Compass Card showing three methods of marking.

Atlantic Ocean, the compass will point west of true north and, in the Pacific, east of true north. The amount of this divergence from true is called variation. As long ago as 1700, Halley constructed charts showing lines of equal variation over the earth. Today we obtain the variation from our charts for any locality. There is also a slow change which is noted on the chart as so much per year and, by figuring from the date of the chart, the present amount can be obtained.

Deviation is a further compass error due to magnetic influences on the individual ship. The intricacies of the subject need not be gone into here. Hard and soft iron, horizontal and vertical iron, permanent, subpermanent and transient magnetism, are all terms used in the discussion and explanation of deviation. It will suffice to understand that, as a ship swings around, its metal and magnetism will be brought into different positions relative to the north point of the compass (which retains its general position) and will exert varying pulls on it, sometimes to one side and sometimes to the other. Deviation varies with latitude.

Heeling error is similar. It occurs when the ship heels or rolls to one side or the other. Iron which has been horizontal approaches the vertical and vice versa. The compass is influenced by this alteration of magnetic force, chiefly when on north and south courses.

Local magnetic disturbance is due to magnetic material outside the ship in the neighborhood. In certain parts of the world (Australia, Labrador, Madagascar, Iceland, the Baltic, Lake Superior) this is a large source of error due to mineral deposits in the ocean or lake bed. Minor causes

are present at docks due to other vessels, metal, etc.

Correction of much of the error which develops with the ship on different headings or due to heeling is done by placing magnets and iron in certain positions near the compass. This is called compass adjusting and should only be done by a thoroughly trained worker. Nothing is done about local magnetic disturbance.

Conversion of true, magnetic, or compass courses, one into another, is easy with the 360° card system. Between true and magnetic, we must know the variation. Between magnetic and compass, we must know the deviation. Bear in mind that a course, however described in these three ways, is the same direction on the earth. When a force pulls the N. point of the compass in a clockwise direction, it is called easterly and if counterclockwise, it is called westerly. If deviation is 5° W. and variation 7° E., the error is the algebraic sum or 2° E. If deviation is 6° W. and variation 2° W., the error is, of course, 8° W.

When thinking for the first time of these disturbances of the compass card, it is a good scheme to think of the card as having just been rotated in a certain way to a certain extent and to imagine yourself pushing it back to its original position. Say you are heading on a compass course of 20° and know that the deviation is 5° E. This means the card has been forced 5° clockwise. Mentally push it back 5° counterclockwise. The heading will now be 25° or the magnetic course. Then suppose you also know the variation is 6° E. This means the card has been forced 6° clockwise. Mentally push it back 6° counterclockwise. The heading will now be 31° or the true course. In each case we have *added*.

Only one rule need be memorized and it is this: *From compass toward true add easterly errors.*

This means that from compass to magnetic we add easterly deviation and from magnetic to true we add easterly variation. Similarly we subtract westerly errors. From true toward compass we do just the opposite: subtract easterly and add westerly. Any one of these can be quickly thought out if we start from the one rule and make the necessary reversals. Examples are shown in Table 7.

TABLE 7
COMPASS ERRORS

Compass	Deviation	Magnetic	Variation	True
64	1 E	65	3 E	68
64	2 W	62	1 W	61
64	3 E	67	4 W	63
64	4 W	60	6 E	66

Methods of determining Deviation.

1. By bearings (azimuth) of the sun. This is the usual method when at sea. Polaris and other stars may also be used.
2. By comparison with a gyro compass.
3. By reciprocal bearings.
4. By bearings of a distant object.
5. By ranges.

The first will be discussed later on in this book under Azimuth. The student is referred to Dutton, Chapter II, for methods 2, 3, and 4. Method 5 which is probably the simplest and most convenient for small craft not at sea, may be briefly outlined as follows:

a. Find some pair of objects in line, one nearer and one farther, easily visible and shown on the chart, and determine the magnetic bearing of the line joining them, from the water where you are located, by means of the "compass rose" on the chart.

b. Send your boat across this line heading first 0°, then 15° and every 15° around the circle, noting the bearing of the range at each separate heading.

c. Make a table showing the above and add a column showing deviation on each heading, as in Table 8.

TABLE 8
FINDING DEVIATION

Ship's Head by Compass	Range by Compass	Range Magnetic	Deviation
0°	60°	64°	4° E
15°	58°	64°	6° E
30°	61°	64°	3° E
45° etc.	65°	64°	1° W

This table of deviations for each compass heading is not sufficient for our needs. It does not tell us, what we are more anxious to know, what compass course to steer in order to make a given magnetic course. We want a table arranged the other way around beginning with equal divisions of magnetic and showing the proper compass course for each.

The Napier Diagram is the means by which the above is accomplished. Details will not be given here but the student is referred to Bowditch or Dutton for a full

explanation. The diagram is basically a series of equilateral triangles each side of a base line. Deviation values for each 15° compass course are plotted and a curve drawn, from which it is easy to obtain the proper compass course for any desired magnetic.

Having constructed the curve as above, a table is made giving the desired compass course for each 15° of magnetic, as in Table 9.

TABLE 9

FOR MAGNETIC—STEER COMPASS

For Magnetic	Steer Compass
15	28
30	41
45	55
60	70
75	84
90, etc.	99

For values between these 15° magnetic intervals it is easy to interpolate. However, for more easily visualizing interpolation, there are two methods by which the table may be diagramed.

First, a double vertical scale may be drawn like two parallel thermometer scales and lines then drawn across from the magnetic to the compass scale indicating equivalent values.

Second, a double compass diagram may be had, with one scale outside the other, the inner representing magnetic and the outer representing compass. Lines again connecting equivalents make the process of conversion and interpolation especially easy.

The Gyro Compass obtains its directive force from the force of the earth's rotation. A full description is given in Bowditch. The essential feature is an electrically driven wheel spinning at 6,000 r. p. m. whose axis seeks to remain in the plane of its meridian. The attachments and mechanical details result in a compass which has the following advantages over the magnetic compass:

a. It is unaffected by the ship's magnetic field. (No deviation.)

b. It seeks the true instead of the magnetic meridian. (No variation.)

c. Its directive force is much greater.

d. It may be located in a safe and central portion of the ship and repeater compasses directed electrically from it may be located anywhere.

The disadvantages are:

a. It is a complex and delicate mechanism.

b. It requires a constant source of electrical power.

c. It requires intelligent and expert care.

d. It is expensive ($2,000 up).

An Azimuth Circle is a ring formed to fit flat over a compass bowl and which can be turned to any desired position. It is graduated from 0° to 360° clockwise. Sighting vanes permit the observer to take bearings of terrestrial objects by turning the circle until the object is in line with the vanes while a reflecting prism throws the compass card into view at the same time. An adjustable dark glass reflector brings celestial bodies into view and a concave mirror and reflecting prism make it easy to take the bearing of the sun.

The Pelorus is a "dumb compass" or card without magnetic needles which can be turned to any desired position, and set. A lubber's line as in the compass marks the direction of the ship's head. Sighting vanes are provided. Peloruses are placed so that views can be had in all directions which is seldom possible with the ship's compass. The card is set to correspond with the heading of the ship by compass and then bearings are taken with it which are the same as compass bearings.

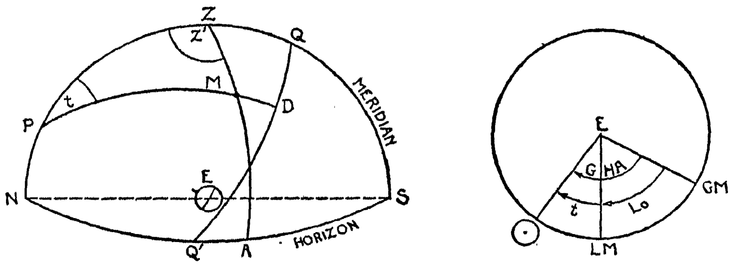
TABLE 10

COMPASS POINTS AND QUARTER POINTS SHOWING EQUIVALENT VALUE IN DEGREES

North to East	East to South	South to West	West to North	Points	D. M. S.
North	East	South	West	0	0° 0′ 00″
N¼E	E¼S	S¼W	W¼N	¼	2° 48′ 45″
N½E	E½S	S½W	W½N	½	5° 37′ 30″
N¾E	E¾S	S¾W	W¾N	¾	8° 26′ 15″
N by E	E by S	S by W	W by N	1	11° 15′ 00″
N by E¼E	ESE¾E	S by W¼W	WNW¾W	1¼	14° 3′ 45″
N by E½E	ESE½E	S by W½W	WNW½W	1½	16° 52′ 30″
N by E¾E	ESE¼E	S by W¾W	WNW¼W	1¾	19° 41′ 15″
NNE	ESE	SSW	WNW	2	22° 30′ 00″
NNE¼E	SE by E¾E	SSW¼W	NW by W¾W	2¼	25° 18′ 45″
NNE½E	SE by E½E	SSW½W	NW by W½W	2½	28° 7′ 30″
NNE¾E	SE by E¼E	SSW¾W	NW by W¼W	2¾	30° 56′ 15″
NE by N	SE by E	SW by S	NW by W	3	33° 45′ 00″
NE¾N	SE¾E	SW¾S	NW¾W	3¼	36° 33′ 45″
NE½N	SE½E	SW½S	NW½W	3½	39° 22′ 30″
NE¼N	SE¼E	SW¼S	NW¼W	3¾	42° 11′ 15″
NE	SE	SW	NW	4	45° 00′ 00″
NE¼E	SE¼S	SW¼W	NW¼N	4¼	47° 48′ 45″
NE½E	SE½S	SW½W	NW½N	4½	50° 37′ 30″
NE¾E	SE¾S	SW¾W	NW¾N	4¾	53° 26′ 15″
NE by E	SE by S	SW by W	NW by N	5	56° 15′ 00″
NE by E¼E	SSE¾E	SW by W¼W	NNW¾W	5¼	59° 3′ 45″
NE by E½E	SSE½E	SW by W½W	NNW½W	5½	61° 52′ 30″
NE by E¾E	SSE¼E	SW by W¾W	NNW¼W	5¾	64° 41′ 15″
ENE	SSE	WSW	NNW	6	67° 30′ 00″
ENE¼E	S by E¾E	WSW¼W	N by W¾W	6¼	70° 18′ 45″
ENE½E	S by E½E	WSW½W	N by W½W	6½	73° 7′ 30″
ENE¾E	S by E¼E	WSW¾W	N by W¼W	6¾	75° 56′ 15″
E by N	S by E	W by S	N by W	7	78° 45′ 00″
E¾N	S¾E	W¾S	N¾W	7¼	81° 33′ 45″
E½N	S½E	W½S	N½W	7½	84° 22′ 30″
E¼N	S¼E	W¼S	N¼W	7¾	87° 11′ 15″
				8	90° 00′ 00″

7. The Astronomical Triangle

NAVIGATION USES an imaginary triangle on the celestial sphere whose three corners are the elevated pole (N. in N. latitude, etc.), the zenith, and the projection of the observed heavenly body. Sometimes we can easily determine the three sides and only need to compute one of the angles. Again we can easily determine two of the sides and one angle and only need to compute the remaining side. These computations are done by spherical trigonometry which will be briefly outlined in the next chapter.



(¼ of Celestial Sphere seen from West) (½ of Celestial Sphere seen from North)

E	Earth	*DM*	Declination			
P	N. Celestial Pole	*MP*	Polar Distance		⊙	Sun
Z	Zenith	*AM*	Altitude	of body.	*LM*	Local Meridian
M	Body Observed	*MZ*	Zenith Distance		*GM*	Greenwich Meridian
QQ'	Celestial Equator	*t*	Hour Angle		*GHA*	Greenwich Hour Angle
QZ	Latitude	*Z'*	Azimuth		*Lo*	Longitude
ZP	Co-Latitude				*t*	Local Hour Angle

FIG. 31. The Astronomical Triangle.

For the present, we will consider what the principal relations are and leave solutions until Part II.

Remember that the celestial triangle is merely a magnification of a terrestrial triangle. Each part, side or angle, or relationship, on the celestial sphere corresponds to the similar unit on the earth. Co-latitude (and hence, of course, latitude), for instance, on the celestial sphere is the same in degrees as that which we may be seeking on earth.

Figure 31 shows on the left a typical astronomical triangle with the parts named. Remember that the angular distance of a quarter circle, no matter of what size, is 90° so that if we know a portion of a quadrant we can find the remaining portion by subtracting the first from 90°. The circular figure on the right shows the celestial sphere as seen from above with projections of the body and two earth meridians.

Table 11 needs little explanation. The first column shows what one starts with; the second shows what is

TABLE 11

FINDING PARTS OF THE ASTRONOMICAL TRIANGLE

Given	Obtain	Procedure	Portion of Triangle Found
Sextant	Altitude (h)	$90° - h$	Zenith Distance (z)
Chronometer	G.C.T.		
G.C.T.	Declination (d)	$90° - d$	Polar Distance (p)*
Naut. Alm.	G.H.A.		
Previous Observation	Latitude (L)	$90° - L$	Co-Latitude (Co-L)
Previous Observation	Longitude (Lo)	G.H.A. $\backsim$ Lo	Local Hour Angle (t)
(As above)	d, L & t	Tables	Azimuth (Z)

* If L and d have opposite names (one N and one S) then $p = 90° + d$.

thereby obtained; the third shows how it is used; and the fourth gives the parts of the astronomical triangle thus found.

Thus 3 sides and 2 angles of the astronomical triangle are sometimes obtainable without the use of formulas. The remaining angle at the observed body is never needed in ordinary navigation.

As the fundamental relationships of celestial navigation are here set forth, this table should be studied with great care and thoroughly understood.

The Celestial Co-ordinator is an ingenious device to give rapid approximate solutions to problems of the astronomical triangle. It consists of a rotating disc on a larger background. The type using the orthographic projection has its under portion printed in black. It is a circle representing observer's meridian with the upper half ruled with horizontal lines for altitude circles every 5°. These are crossed by elliptical arcs from the zenith for azimuth circles every 5°. Over this and fastened to it at the center is a transparent circular disc ruled similarly but in red and in both halves. Its border represents observer's meridian. A straight line across it is for the equinoctial. Crossing this at right angles is the polar axis. Lines parallel to the equinoctial are declination circles every 5°. Elliptical arcs from the poles represent hour circles at 3¾° (15 minutes of time) intervals (in the type produced by the N. Y. Power Squadron). In using the co-ordinator the upper part is set according to observer's latitude. Then various combinations of problems involving h, t, d, and Z may be solved. The greatest usefulness appears in star work. Given t and d, one gets the approximate h and Z which shows where to look for the star. Given h and Z of an unknown observed star, one gets d and t. From t and known longitude, one gets G.H.A. With d and G.H.A. and the Nautical Almanac, the star can be identified.

8. Trigonometry

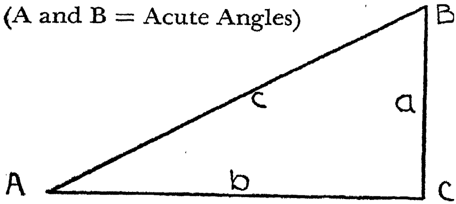
NAUTICAL ASTRONOMY makes constant use of trigonometry. The commonest example of this is the need for computing altitude for comparison with a sextant observation. This and various other needs will be explained in Part II. A knowledge of this branch of mathematics is not essential to the navigator but some familiarity with its fundamental concepts is highly desirable. Therefore, both for the benefit of the novice and as a mind refresher for those who at some distant past date have studied trigonometry, the following pages are provided. They contain enough to indicate the general features of the subject and to suggest how various formulas, which will appear later, may have been derived. Remember that in plane trigonometry, only angles are measured in degrees. In spherical trigonometry not only angles but also sides are so measured. The reader is referred for further details to any modern text on the subject. An excellent one is Palmer and Leigh: *Plane and Spherical Trigonometry,* 4th edition (McGraw Hill Book Co., Inc., New York and London, 1934).

Plane Trigonometry

This branch of trigonometry investigates the relations that exist between the parts of triangles which lie in a plane.

TABLE 12
DEFINITION OF THE TRIGONOMETRIC FUNCTIONS OF PLANE RIGHT TRIANGLES

(A and B = Acute Angles)



Sin (sine) $A = \dfrac{\text{side opposite}}{\text{hypotenuse}} = \dfrac{a}{c} = \cos B$ ($\sin A$ = always < 1)

Cos (cosine) $A = \dfrac{\text{side adjacent}}{\text{hypotenuse}} = \sin B$ ($\cos A$ = always < 1)

Tan (tangent) $A = \dfrac{\text{side opposite}}{\text{side adjacent}} = \dfrac{a}{b} = \cot B \begin{cases} \tan A \text{ under } 45° = \text{always} < 1 \\ \tan A \text{ over } 45° = \text{always} > 1 \end{cases}$

Cot(cotangent)$A = \dfrac{\text{side adjacent}}{\text{side opposite}} = \dfrac{b}{a} = \tan B \begin{cases} \cot A \text{ under } 45° = \text{always} > 1 \\ \cot A \text{ over } 45° = \text{always} < 1 \end{cases}$

Sec (secant) $A = \dfrac{\text{hypotenuse}}{\text{side adjacent}} = \dfrac{c}{b} = \csc B$ ($\sec A$ = always > 1)

Csc (cosecant) $A = \dfrac{\text{hypotenuse}}{\text{side opposite}} = \dfrac{c}{a} = \sec B$ ($\csc A$ = always > 1)

HAVERSINES

1-cos A is called versed sin A (vers A)

½ (1-cos A) is called haversine A (hav A)

By formula (#22 in Palmer & Leigh):

Sin ½ $A = \sqrt{\dfrac{1-\cos A}{2}}$ which, being squared, becomes:

$\sin^2 \frac{1}{2} A = \frac{1}{2}(1-\cos A)$. Hence:—

Haversine of an angle = square of the sine of ½ the angle.

TABLE 13

EQUIVALENTS OF THE TRIGONOMETRIC FUNCTIONS OF ONE ACUTE ANGLE OF A PLANE RIGHT TRIANGLE WHEN HYPOTENUSE = 1

$$\operatorname{Sin} A = \tan A \cos A = \frac{1}{\csc A} = \frac{\tan A}{\sec A} = \frac{\cos A}{\cot A} = \sqrt{1 - \cos^2 A} = \frac{\tan A}{\sqrt{1 + \tan^2 A}} = \frac{1}{\sqrt{1 + \cot^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\operatorname{Cos} A = \cot A \sin A = \frac{1}{\sec A} = \frac{\cot A}{\csc A} = \frac{\sin A}{\tan A} = \sqrt{1 - \sin^2 A} = \frac{\cot A}{\sqrt{1 + \cot^2 A}} = \frac{1}{\sqrt{1 + \tan^2 A}} = \frac{\sqrt{\csc^2 A - 1}}{\csc A}$$

$$\operatorname{Tan} A = \sin A \sec A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \frac{\sec A}{\csc A} = \sqrt{\sec^2 A - 1} = \frac{\sin A}{\sqrt{1 - \sin^2 A}} = \frac{1}{\sqrt{\csc^2 A - 1}} = \frac{\sqrt{1 - \cos^2 A}}{\cos A}$$

$$\operatorname{Cot} A = \cos A \csc A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \frac{\csc A}{\sec A} = \sqrt{\csc^2 A - 1} = \frac{\cos A}{\sqrt{1 - \cos^2 A}} = \frac{1}{\sqrt{\sec^2 A - 1}} = \frac{\sqrt{1 - \sin^2 A}}{\sin A}$$

$$\operatorname{Sec} A = \csc A \tan A = \frac{1}{\cos A} = \frac{\csc A}{\cot A} = \frac{\tan A}{\sin A} = \sqrt{1 + \tan^2 A} = \frac{\csc A}{\sqrt{\csc^2 A - 1}} = \frac{1}{\sqrt{1 - \sin^2 A}} = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

$$\operatorname{Csc} A = \sec A \cot A = \frac{1}{\sin A} = \frac{\sec A}{\tan A} = \frac{\cot A}{\cos A} = \sqrt{1 + \cot^2 A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}} = \frac{1}{\sqrt{1 - \cos^2 A}} = \frac{\sqrt{1 + \tan^2 A}}{\tan A}$$

The sum of the angles of a plane triangle = 180°.

The complement of an angle = 90° minus the angle.

The supplement of an angle = 180° minus the angle.

When one quantity so depends on another that for every value of the first there are one or more values of the second, the second is said to be a function of the first.

Connected with any angle there are six ratios that are of fundamental importance, as upon them is founded the whole subject of trigonometry. They are called the trigonometric functions of the angle. To each and every angle there corresponds but one value of each trigonometric function.

Tables 12, 13, 14, 15 will show the nomenclature used and some of the fundamental relationships of the functions of, first, acute angles and, second, angles of any size.

Spherical Trigonometry

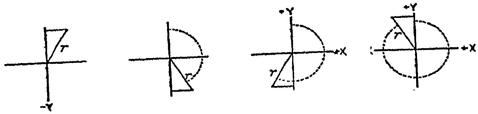
This branch of trigonometry investigates the relations that exist between the parts of a spherical triangle.

A spherical triangle is the figure on the surface of a sphere bounded by three arcs of great circles. The three arcs are the sides of the triangle, and the angles formed by the arcs at the points where they meet are the angles of the triangle. The angle between two intersecting arcs is measured by the angle between the tangents drawn to the arcs at the point of intersection. The sum of the sides of a spherical triangle is less than 360°. The sum of the angles of a spherical triangle is greater than 180° and less than 540°. In a spherical triangle there are six parts, three sides and three angles, besides the radius of the sphere

TABLE 14

TRIGONOMETRIC FUNCTIONS OF ANY ANGLE WITH THE SIGN FOR EACH QUADRANT

(The sign of the hypotenuse or distance is always +)



ANGLE IN QUADRANT	I	II	III	IV
Sin $= \frac{x}{r}$	+	+		
Cos $= \frac{y}{r}$	+			+
Tan $= \frac{x}{y}$	+		+	
Cot $= \frac{y}{x}$	+		+	
Sec $= \frac{r}{y}$	+			+
Csc $= \frac{r}{x}$	+	+		

NOTE: The angles here and in Figure 33 have been drawn increasing clockwise (in contrast to Figure 137 in Bowditch) so as to conform to the 360° compass and its quadrants.

which is supposed known. In general, if three of these parts are given, the other parts can be found.

A right spherical triangle is one which has an angle equal to 90°. In such a triangle, two given parts in addition to the right angle are sufficient to solve the triangle.

Napier's Rules of Circular Parts. In a right spherical triangle, omitting the right angle, consider the two sides (a and b) adjacent to the right angle, the complements of the two other angles (co-A and co-B) and the complement

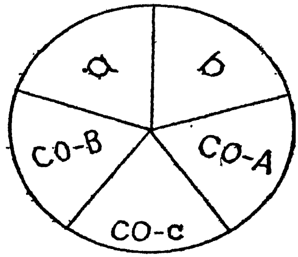


FIG. 32. For Napier's Rules.

of the remaining side (co-c). Arrange these in a circle as in Figure 32. Any one of these five parts may be selected and called a *middle* part; then the two parts next to it are called *adjacent* parts and the other two parts, *opposite* parts. Napier's rules are:

1. The sine of a middle part equals the product of the tangents of the adjacent parts.

2. The sine of a middle part equals the product of the cosines of the opposite parts.

The ten formulas for solution of a right spherical triangle can be derived from these two rules.

TABLE 15

EQUIVALENT TRIGONOMETRIC FUNCTIONS OF ANGLES IN THE DIFFERENT QUADRANTS

(A = Acute)

(See Fig. 33)

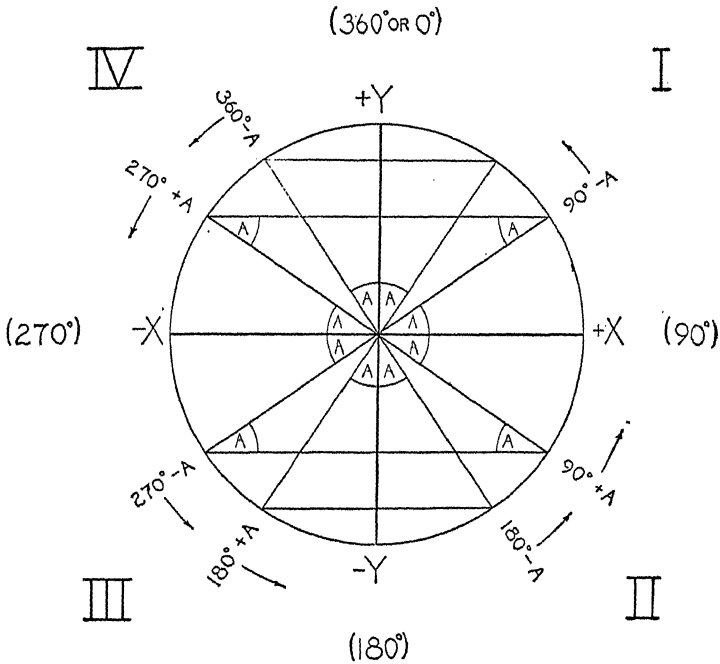
Quadrant	ALL ITEMS IN EACH VERTICAL COLUMN ARE EQUAL					
I	$\sin A$	$\cos A$	$\tan A$	$\cot A$	$\sec A$	$\csc A$
	$\cos\ (90^\circ - A)$	$\sin\ (90^\circ - A)$	$\cot\ (90^\circ - A)$	$\tan\ (90^\circ - A)$	$\csc\ (90^\circ - A)$	$\sec\ (90^\circ - A)$
II	$-\cos\ (90^\circ + A)$	$\sin\ (90^\circ + A)$	$-\cot\ (90^\circ + A)$	$-\tan\ (90^\circ + A)$	$\csc\ (90^\circ + A)$	$-\sec\ (90^\circ + A)$
	$\sin\ (180^\circ - A)$	$-\cos\ (180^\circ - A)$	$-\tan\ (180^\circ - A)$	$-\cot\ (180^\circ - A)$	$-\sec\ (180^\circ - A)$	$\csc\ (180^\circ - A)$
III	$-\sin\ (180^\circ + A)$	$-\cos\ (180^\circ + A)$	$\tan\ (180^\circ + A)$	$\cot\ (180^\circ + A)$	$-\sec\ (180^\circ + A)$	$-\csc\ (180^\circ + A)$
	$-\cos\ (270^\circ - A)$	$-\sin\ (270^\circ - A)$	$\cot\ (270^\circ - A)$	$\tan\ (270^\circ - A)$	$-\csc\ (270^\circ - A)$	$-\sec\ (270^\circ - A)$
IV	$\cos\ (270^\circ + A)$	$-\sin\ (270^\circ + A)$	$-\cot\ (270^\circ + A)$	$-\tan\ (270^\circ + A)$	$-\csc\ (270^\circ + A)$	$\sec\ (270^\circ + A)$
	$-\sin\ (360^\circ - A)$	$\cos\ (360^\circ - A)$	$-\tan\ (360^\circ - A)$	$-\cot\ (360^\circ - A)$	$\sec\ (360^\circ - A)$	$-\csc\ (360^\circ - A)$

Table 15 leads to the following rules which are useful in certain "Sailings" problems which will be found in Part III

1. The value of any function of any angle in the three higher quadrants is always equal to the value of the same function of some angle in the first quadrant, as:

Given Angle in Quadrant	*Corresponding Angle in Quadrant I*
II	180° − given angle
III	given angle − 180°
IV	360° − given angle

2. To find an angle, known to be in a higher quadrant, for a log in Table 33, Bowditch, take out the angle in Quadrant I and treat as follows:



(To be used with Table 15)

FIG. 33. Angles in the Different Quadrants.

The *A* angles at center are constructed equal. Each *A* angle near circumference equals *A* since its horizontal side is parallel, by construction, to the *X* axis and, by geometry, a straight line (here a radius) connecting two parallel lines makes an angle on one side with the first line which is equal to the angle it makes on the other side with the second line.

For Quadrant	*Use*
II	$180° -$ angle in I
III	$180° +$ angle in I
IV	$360° -$ angle in I

Oblique Spherical Triangles are solved by formulas derived from the following theorems:

1. Sine theorem. In any spherical triangle, the sines of the angles are proportional to the sines of the opposite sides.

2. Cosine side theorem. In any spherical triangle, the cosine of any side is equal to the product of the cosines of the other two sides, increased by the product of the sines of these sides times the cosine of their included angle.

3. Cosine angle theorem. The cosine of any angle of a spherical triangle is equal to the product of the sines of the two other angles multiplied by the cosine of their included side, diminished by the product of the cosines of the two other angles.

Examples:

$$(1)\quad \frac{\sin A}{\sin a}=\frac{\sin B}{\sin b}=\frac{\sin C}{\sin c}$$

$$(2)\quad \cos a=\cos b\cos c+\sin b\sin c\cos A$$

$$(3)\quad \cos A=\sin B\sin C\cos a-\cos B\cos C$$

The following are useful haversine formulas for any spherical triangle:

For an angle when the sides are given:

$$\operatorname{hav} A=\frac{\operatorname{hav} a-\operatorname{hav}(b\sim c)}{\sin b\sin c}$$

For a side when other sides and included angle are given:

$$\text{hav } a = \text{hav } (b \backsim c) + \text{hav } \theta$$

where $\text{hav } \theta = \text{hav } A \sin b \sin c$

9. Logarithms

TRIGONOMETRIC FORMULAS usually call for multiplication of various long numbers. To multiply one cosine by another and this perhaps by a haversine would be a tedious process with old style arithmetic. By the use of logarithms, the processes of multiplication, division, raising to a power, and extracting a root of arithmetical numbers are usually much simplified.

If $N = b^x$, then $x =$ the logarithm of N to the base b.

The logarithm of a number to a given base is the exponent by which the base must be affected to produce that number.

As $100 = 10^2$ so $\log_{10} 100 = 2$, or the logarithm of 100 to the base 10 is 2. This is abbreviated to $\log 100 = 2$.

Logarithms were invented by Lord Napier of Scotland (1550-1617) and described by him in 1614. He used the base 2.7182818 called "e." This system is used in advanced and theoretical work.

Prof. Briggs of London (1556-1631) modified the above by using the base 10. This system is called the common or Briggs system and is the one usually used in computing and is given in the 1938 Bowditch, Table 32 for logs of numbers and Table 33 for logs of trigonometric functions of angles.

The word "natural" is used to distinguish from the log-

arithm. Table 31 gives natural, that is, actual values of, sines, cosines, tangents and cotangents for all angles to 90°. Table 33 is where one will find the log sines and log cosines, as well as the logs of the other four trigonometric functions, of angles to 180°. Table 34 gives both natural haversines and log haversines of angles to 360°. (Although omitted to save space, −10 belongs after every log in Tables 33 and 34.)

Multiplication of two numbers is accomplished by adding their logs and then finding the number of which this sum is the log.

Division is accomplished by subtracting the log of the divisor from the log of the dividend and then finding the number of which this result is the log.

Raising to a power is done by multiplying the log of the given number by the index of the power and then finding the number of which this product is the log.

Extracting a root is done by dividing the log of the given number by the index of the root and then finding the number of which this quotient is the log.

The student should consult Bowditch, pp. 324-26, for details.

While rules are given for obtaining logs of numbers and numbers for logs, I believe the rules on the next two pages will be found somewhat simpler.

Table 16 is to show examples of logs with various characteristics (numbers to left of decimal point) for one mantissa (number to right of decimal point).

To Find Log of a Number

Characteristic:

For number $> 1 = 1 <$ the number of figures including zeros to left of number's decimal point.

For number $< 1 = 9 -$ the number of zeros directly to right of number's decimal point and -10 to right of mantissa.

Mantissa:

Disregard decimal point of number when looking it up in Table 32 Bowditch.

For number of 1 figure: Add 2 zeros to right and treat as 3 figures.

For number of 2 figures: Add 1 zero to right and treat as 3 figures.

For number of 3 figures: Find number in left column and mantissa opposite it in column headed 0.

For number of 4 figures: Find first 3 in left column and mantissa opposite them in column headed by 4th figure.

For number of 5 or more figures: Find mantissa for first 4. Then *add* to it the following: figure in column d $\times$ remaining figure or figures, first pointing off as many places as there are remaining figures and disregarding fraction or, if it exceeds .5, raising total to next number.

To Find Number for a Log

May find exact mantissa in Table 32 Bowditch and take out first 3 figures from left column and 4th from top.

If mantissa lies between 2 given in table:

Take 4 figure number of next lower mantissa.

Note difference between next lower and given mantissa (1st diff.).

Note difference between next lower and next higher mantissa (col. d).

Note P. P. table under figure found for d.

Note 1st diff. figure in right column.

Note figure on same line in left column. This is 5th figure of number.

If over 5 figures in number (as when characteristic is 5 and number therefore has 6 figures to left of decimal point), get 4 figure number for next lower mantissa and then use equation:

$$\frac{\text{1st diff.}}{\text{d}} = \text{5th and succeeding figures. (Always} < 1.)$$

Disregard decimal point but retain any zeros that may come between it and other figures and add 1 or more zeros on right if characteristic calls for more figures in number.

If number is > 1: Place point to right of 1 more figures than number of characteristic.

If number is < 1 (log ending in -10 with characteristic 9 or less): Subtract characteristic from 9 and add that many zeros before figures already found and put point to left.

TABLE 16

EXAMPLES OF LOGARITHMS

Number	Logarithm		
12,345,678,912.	10.09152	or	20.09152 − 10
1,234,567,891.2	9.09152		19.09152 − 10
123,456,789.12	8.09152		18.09152 − 10
12,345,678.912	7.09152		17.09152 − 10
1,234,567.8912	6.09152		16.09152 − 10
123,456.78912	5.09152		15.09152 − 10
12,345.678912	4.09152		14.09152 − 10
1,234.5678912	3.09152		13.09152 − 10
123.45678912	2.09152		12.09152 − 10
12.345678912	1.09152		11.09152 − 10
1.2345678912	0.09152		10.09152 − 10
.12345678912	− 1.09152		9.09152 − 10
.012345678912	− 2.09152		8.09152 − 10
.0012345678912	− 3.09152		7.09152 − 10
.00012345678912	− 4.09152		6.09152 − 10
.000012345678912	− 5.09152		5.09152 − 10
.0000012345678912	− 6.09152		4.09152 − 10
.00000012345678912	− 7.09152		3.09152 − 10
.000000012345678912	− 8.09152		2.09152 − 10
.0000000012345678912	− 9.09152		1.09152 − 10
.00000000012345678912	− 10.09152		0.09152 − 10
.000000000012345678912	− 11.09152		9.09152 − 20

Part II: Procedures

10. Introduction to Position Finding

SOLUTIONS FOR LATITUDE, as such, are not so important in the modern practice of navigation as they once were. The same can be said of longitude. This is because, as we shall soon see, the newer navigation obtains from one observation a "line of position," on which the ship is situated, and crosses this with another line from another observation thus obtaining a "fix" on the chart, from which the latitude and longitude can then be read off giving the exact position. Discussion and argument still go on between navy and merchant marine on the merits of the newer methods. Inasmuch as the whole art and science of position finding has evolved through first obtaining latitude and longitude and because many officers of the merchant marine still depend on these older methods and must use them in their examinations for promotion, it seems well to briefly explain them to the beginner.

One of the first things to understand is the *nautical mile*. It is defined in the U. S. A. as being 6,080.27 feet in length, equal to $1/60$ part of a degree, or 1 minute of arc, of a great circle of a sphere whose surface is equal in area to the area of the surface of the earth. The earth is somewhat flattened at the poles which slightly alters the length of 1 minute high on a meridian. This, however, is disregarded in navigation and a change of 1 minute of latitude always is taken to mean a change of 1 nautical mile north

or south. Since the meridians converge toward the poles, the difference of longitude produced by a change of position of 1 mile to the east or west will increase with the latitude. For instance, 1 mile on the equator will cause a change of longitude of 1 minute while at latitude 60° it will cause a change of 2 minutes.

Before doing any actual work, the amateur navigator must understand the principles of the *Mercator chart.* (Gerardus Mercator, Flemish cartographer, 1512-1594.) The transfer of a spherical surface, such as a globe map of the world, onto a flat surface, presents many difficulties. The portion of the globe between Lat. 60° N. and Lat. 60° S. may, however, be transferred by placing a cylinder of transparent paper around the globe, tangent at the equator, and projecting onto it the features of the globe as seen from its center. Cutting this cylinder vertically at some point and laying it out flat will show the meridians of longitude not converging but as parallel vertical lines. The parallels of latitude will be horizontal parallel lines but farther apart the farther away they are from the equator. As the distance between meridians becomes more and more in excess of the true proportional distance, the increasing distance between parallels makes up for it and proportion in a given region is maintained. (This is not literally accomplished as described but is done mathematically. See Dutton, Chap. I.) Of course, there is great distortion of areas in high latitudes. The great advantage of a chart on this basis is that any course which cuts successive meridians at the same angle becomes a straight line. If it were cutting meridians at some angle other than 90° and drawn on a globe, it would have to be a curve of a spiral

form. Such a curve is called a *Rhumb Line.* Scales of miles as on ordinary maps are not possible here but, since a minute of latitude equals a nautical mile, one uses the latitude scale marked at the side of the chart at the level of the region measured.

Position plotting sheets are blank charts made on the Mercator system and important for the navigator in the graphic solution of problems. One series issued by the Hydrographic Office is of 12 sheets covering latitudes from 0° to 60° and can be used for north or south of the equator. The price is 20 cents a sheet, sold singly for a 5° area. Their use saves the mutilation of charts, and they are large enough (about 4 x 3 ft.) to permit the recording of sufficient data for a good record. The parallels of latitude are, of course, numbered, but the meridians of longitude are not. The user numbers the latter according to his location. A smaller and more convenient size (26 x 19 inches) is issued in 16 sheets covering latitudes 0°-49° and sells for 10 cents a sheet.

Tables of logarithms of trigonometric functions are necessary for the solution of various equations which will appear in the descriptions of the several procedures. Bowditch (H. O. 9) contains all the essential tables. Among the many newer systems which have been devised for position finding, H. O. 211 by Comdr. Ageton is the one preferred by the present writer. Reasons for this will be given in the chapter on Short-Cut Systems. Meanwhile, equations will generally be presented in two forms. Both are solvable by the Bowditch tables but the second in each case is especially designed for solution by H. O. 211.

The several procedures will first be briefly outlined with the idea that the student should get a rapid survey of the

general principles before getting into the details of individual problems. In Part III will be found examples worked out for each of the more important procedures.

The expression *"dead reckoning"* which will frequently be used comes from deduced (ded.) reckoning and is abbreviated to D. R. It is the method of finding a ship's position by keeping track of courses steered and distances run from the last well-known position. This is explained in Chapter 21.

Throughout the following discussions of procedures, preference will be given to the new methods in which G. H. A. is taken from N. A.

11. Latitude

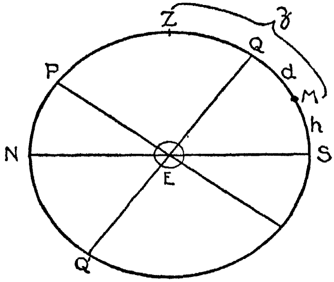
THE LATITUDE of a place is the arc of the meridian of the place subtended between the equator and the place. It is labeled north or south in relation to the equator. It may also be described as the angular distance on the celestial sphere along the hour circle of the place between the equinoctial and the projection of the place, or its zenith. By geometry, it also equals the altitude of the elevated pole.

Figure 34 shows the four cases which cover all latitude calculations from observations of a body on the meridian. It is important to study these and understand the equations, which may be summarized as follows:

1. L & d opposite names:	$L = z - d$
2. L & d same name & $L > d$:	$L = z + d$
3. L & d same name & $d > L$:	$L = d - z$
4. L & d same name, lower transit:	$L = 180° - (d + z)$
	$= h + (90° - d)$
	$= h + p$

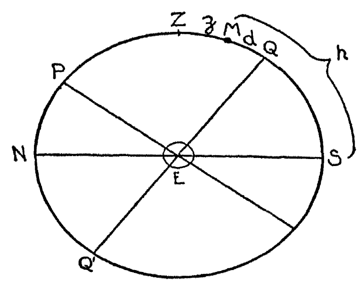
Regardless of what body is used, these principles apply. The next problem is to determine just when the body is on the meridian. There are three ways:

1. Measure with sextant the altitude of a body about to make an upper transit, that is, crossing the meridian from east to west, and continue to measure it at short intervals noting the time of each observation, until the altitude



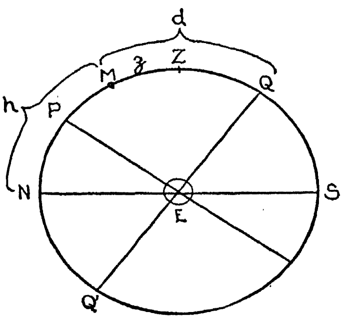
1

$L = (90° - h) - d = z - d$

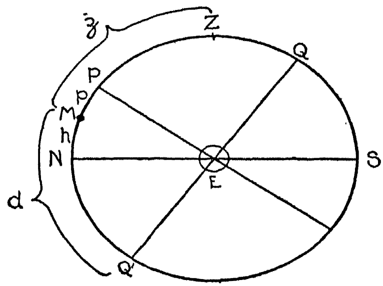


2

$L = (90° - h) + d = z + d$



$L = d - (90° - h) = d - z$



$$\begin{aligned} L &= 180° - [d + (90° - h)] \\ &= 180° - (d + z) \\ &= h + (90° - d) = h + p \end{aligned}$$

Each Big Circle = Projection of Celestial Sphere on Plane of the Meridian

E	= Earth	M	= Projection of Body
P	= Elevated Pole	h	= Altitude
Z	= Zenith	z	= Zenith Distance
QQ'	= Equinoctial	d	= Declination
NS	= Horizon	p	= Polar Distance

$\left.\begin{matrix} QZ \\ NP \end{matrix}\right\}$ = Latitude

FIG. 34. The Four Cases of Latitude from Meridian Altitude Observation.

begins to decrease. Then take the greatest altitude as the meridian altitude. This is not especially accurate, but is often used for the sun.

2. Observe the true bearing of the body and measure its altitude when it is directly south or north as the case may be. With a good compass, steady ship and not too high altitude, this gives fair results.

3. Calculate in advance the time of transit. This is the most dependable method. It necessitates knowing the correct longitude if ship is stationary or moving true north or south. If ship is making any progress east or west, the rate of longitude change must be known in addition to the longitude and time at the start of the calculation. The usual method of making this calculation for apparent noon is known as Todd's and was devised by him when a midshipman at Annapolis (*see* Bowditch or Dutton). Tables based on his equations have been published as H. O. 202 (Noon-Interval Tables).

Latitude by Noon Sun

The G. C. T. of apparent noon may be found by the following modification of Todd's method, using G. H. A. without looking up Eq. T. No mention is made of watch time or its difference from chronometer as this becomes very confusing to the beginner and can easily be dispensed with on smaller boats either by taking observations with chronometer nearby or by setting a stop-watch exactly with chronometer. The true sun is here assumed to be "moving" at the same speed as the mean (civil time) sun. The error of combining a mean time interval with a civil time interval is immaterial (about 1 second per hour).

Interval to Noon and G. C. T. of Local Apparent Noon

1. G. C. T. at an instant in A. M. when longitude is known.
2. G. H. A. of sun in arc from N. A. for this instant.
3. Combine G. H. A. with longitude obtaining *t in arc, always E.*
4. Convert this arc into time. This is *interval to apparent noon at known longitude.*
5. If ship is not changing longitude, add *t* in time (#4) to G. C. T. (#1), obtaining *G. C. T. of Local Apparent Noon.*
6. If ship is changing longitude, subtract *t* in time (#4) from 12, obtaining *L. A. T. of beginning of interval.*
7. Bowditch, Table 3: Enter with course in degrees (top or bottom of page) and speed in knots (col. labelled Dist.), obtaining *miles made E. or W. in 1 h.* (col. labelled Dep.).
8. Bowditch, Table 3: Enter with latitude of ship (figure for course in degrees, top or bottom of page) and miles made E. or W. in 1 h. (col. labelled Lat.), obtaining *change of longitude E. or W. in minutes of arc per hour* (col. labelled Dist.).
9. H. O. 202 (Noon-Interval Tables): Enter with L. A. T. of beginning of interval (#6) and change of longitude E. or W. in minutes of arc per hour (#8), obtaining *interval to apparent noon* with ship maintaining course and speed.
10. Add this interval to G. C. T. (#1), obtaining G. C. T. of L. A. N.

Another and simpler method but which requires charting is that of Commander Weems. Put in my own words it is as follows:

Weems' Method for Interval to Noon and Z. T. or G. C. T. of L. A. N.

1. At some time in morning determine sun's *t* east.

2. Convert this arc to time and fraction of hour to decimal. (This is time sun needs to reach meridian of #1 position.)

3. On chart from position of #1 run line for course as far as speed for time of #2 would take ship.

4. Find *DLo* in minutes for length of this line. If not due E. or W., drop perpendiculars from each end to middle latitude and measure on that.

5. Since sun moves W. 15′ per minute of time, divide *DLo* of #4 by 15′ to get time in minutes that sun would require to cover this *DLo*.

6. If course is easterly, this time is *saved,* so subtract it from time of #2 for interval to noon (sun on ship's meridian).

7. If course is westerly, this time is *lost,* so add it to time of #2 for interval to noon.

8. Apply this interval time to Z. T. or G. C. T. of #1 to get Z. T. or G. C. T. for noon sight at L. A. N.

The Meridian Sight

Whatever method of finding the proper time for the meridian observation is used, the following steps must then be carried out:

1. Take sextant altitude of body.
2. Note G. C. T.
3. Make usual altitude corrections.
4. Subtract from 90° to get z.
5. Find declination in N. A. for G. C. T. of observation.
6. Combine d and z according to which of the four cases was present and obtain latitude.

Reduction to the Meridian

A small cloud may spoil the actual meridian sight. Observations are therefore often taken any time within 28 minutes of noon, before or after, because such can be "reduced" to the meridian by Tables 29 and 30 in Bowditch. This procedure is based on the following equation:

Meridian altitude = corrected altitude at observation + (change of altitude in 1 minute of time from meridian × square of time interval from meridian passage) or $H = h_o + a\,t^2$.

Entering Table 29 with D.R. latitude and declination, obtain a.

Entering Table 30 with this a and the time of observation from meridian passage (given for ½ minute), obtain $a\,t^2$.

This $a\,t^2$ is to be added to a corrected altitude observed near upper transit (or subtracted from a corrected altitude

observed near lower transit) to obtain the corrected altitude at meridian passage.

Having thus found the meridian altitude, it is treated as explained under Meridian Sight to obtain latitude. This latitude is that of the vessel at the instant of observation. The latitude at noon will depend on the run between noon and sight.

Latitude by Star, Planet and Moon Transits

Latitude may also be found by measuring the altitude of some body, other than the sun, at meridian transit. This procedure is not generally recommended. The time for observation at twilight is quite limited and, while waiting for a given transit, the horizon or body may fade. So, only the one co-ordinate, latitude, is obtainable. Hence, the usual practice is to measure two or more stars whose bearings differ by about 90°, from which, as will be explained later, the exact position of the ship can be found.

The general principle of finding G. C. T. of transit is this: Longitude must be known. When body is crossing local meridian, its G. H. A. = the longitude if west, or 360° minus the longitude if east. Hence find from N. A. at what G. C. T. the body will have G. H. A. equal to the longitude, or to 360° − the longitude. This will be G. C. T. of transit. Details of a new uniform procedure will be found in Part III, Chapter 30, Problems, Transit.

Latitude by Polaris

The altitude of the elevated pole of the celestial sphere

equals the latitude. So in the northern hemisphere, it is convenient to measure the altitude of the north star or Polaris which is close to the celestial pole and, by means of certain tables in the N. A., "reduce" this altitude to that of the actual celestial pole. Polaris has an apparent motion counterclockwise in a circle with radius of about 1° around the actual pole. The steps in the process to thus find latitude are as follows (references are to 1943 N. A.):

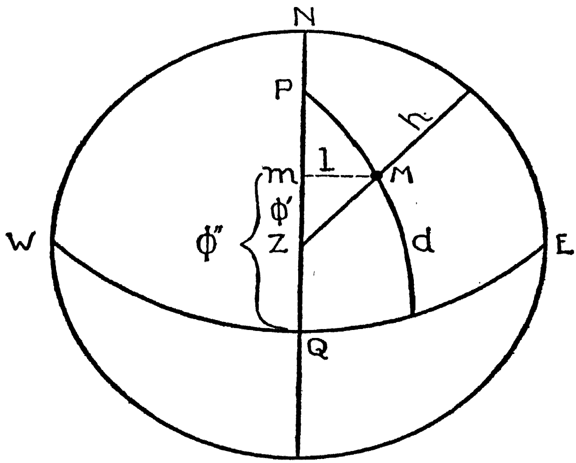
1. Take sextant altitude of Polaris.
2. Note G. C. T. and longitude.
3. Make usual altitude corrections.
4. N. A. table for G. H. A. of Polaris (W) (p. 280) at 0^h G. C. T. of date.
5. N. A. Table "Correction to be added to tabulated G. H. A. of stars" (pp. 214-16) using G. C. T. and obtaining correction.
6. Add results of #4 and #5, subtracting 360° if over 360°, obtaining G. H. A. at G. C. T. of observation.
7. Apply longitude to this G. H. A. obtaining L. H. A. of Polaris (W).
8. N. A. Table III (p. 284) entering with L. H. A. and obtaining correction for L. H. A. to be applied to true altitude.
9. Add this to or subtract it from true altitude obtaining approximate latitude.

Latitude by Phi Prime, Phi Second

This old method for determining latitude from (1) a single altitude of a body not on the meridian, (2) G. C. T.

of the observation, and (3) known longitude, is now seldom used, but is given here as an example of one of the phases of navigation which preceded the Saint-Hilaire method.

The method is best restricted to conditions where the body is within 3 hours of meridian passage, of declination at least 3°, and not over 45° from the meridian in azimuth. This last means that a line from body to zenith should not make an angle of over 45° with meridian. (*See* FIG. 35.)



Projection of the Celestial Sphere on the Plane of the Horizon.

WQE = Equator
Z = Zenith
P = Elevated Pole
M = Body
d = Declination
h = Altitude
Mm = Perpendicular from Body to Meridian = l
$\phi' = mZ$ = Zenith distance of m
$\phi'' = mQ$ = Declination of m
QZ = Latitude to be found

FIG. 35. Latitude by Phi Prime, Phi Second.

One of the two sets of equations given in Bowditch (1933) for the solution is as follows:

$$\begin{aligned} \sin l &= \cos d \sin t \\ \sin \phi'' &= \sin d \sec l \\ \cos \phi' &= \sin h \sec l \end{aligned}$$

Give ϕ'' same name as d.

Mark ϕ' North if body bears north and east or north and west.

Mark ϕ' South if body bears south and east or south and west.

Combine ϕ' and ϕ'' by adding, if different names or subtracting, if same.

The result will be latitude, except in the case of bodies nearer lower transit when $180° - \phi''$ must be substituted for ϕ''. (The rules for marking ϕ' and for combining are Bowditch's reversed, in order to make the process conform to other procedures in general use.)

As the author has pointed out (U. S. N. I. P. Sept. 1935), these Bowditch equations can be converted for satisfactory use with H. O. 211 substituting R for l as follows:

$$\csc R = \csc t \sec d$$

$$\csc \phi'' = \frac{\csc d}{\sec R}$$

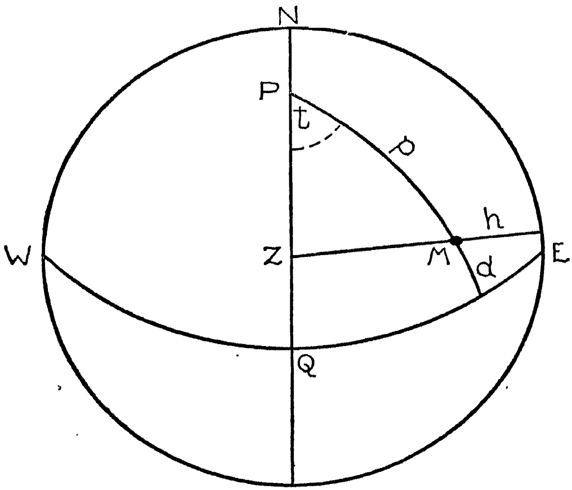
$$\sec \phi' = \frac{\csc h}{\sec R}$$

Summary

1. Take sextant altitude of body.
2. Note G. C. T. and longitude.
3. Make usual altitude corrections.
4. N. A. for declination and G. H. A.
5. Combine G. H. A. and longitude for *t*.
6. Solve by 211 method for latitude.

12. Longitude and Chronometer Error

THE LONGITUDE of a place is the arc of the equator intercepted between the prime meridian (Greenwich, England) and the meridian of the place, measured from the prime meridian toward the east or west through 180°.



Projection of the Celestial Sphere on the Plane of the Horizon.

WQE = Equator	p = Polar distance
Z = Zenith	h = Altitude
P = Elevated Pole	QZ = Latitude
M = Body	t = Local hour angle to be found
d = Declination	

FIG. 36. Longitude by Time Sight.

The "Time Sight" method for longitude dates back to 1763 and is still much in use in the merchant service though not in the navy. It calls for (1) a single altitude of a body preferably near the prime vertical (bearing east or west), (2) G. C. T. of the observation and (3) known latitude. Practically, the body should be between 3 and 5 hours of meridian passage. (*See* FIG. 36.)

The long-used equation for this problem is:

$$\text{hav } t = \sec L \csc p \cos s \sin (s - h)$$

where $s = \frac{1}{2}(h + L + p)$

Having thus found t, it is combined with the G. H. A. from N. A. to give longitude.

Hulbert Hinkel, Jr. (U. S. N. I. P. April 1935) presented the following substitute equation for use with H. O. 211:

$$\csc^2 \tfrac{1}{2} t = \frac{\sec s \csc (s - h)}{\sec L \csc p}$$

Summary

1. Take sextant altitude of body.
2. Note G. C. T. and latitude.
3. Make usual altitude corrections.
4. N. A. for declination and G. H. A.
5. Solve by 211 method for t.
6. Combine t with G. H. A. for longitude.

NOTE: A method for finding the time at which the sun will be on the prime vertical will be found in Part III, Chap. 30, Problems.

Chronometer Error

Radio time signals now furnish the best means of determining chronometer error at sea.

In some circumstances, however, the radio receiver may fail, or there may be no radio. In such case, if correct longitude is known, the process is as follows:

1. Take sextant altitude of body as near prime vertical as possible.
2. Note chronometer (approx. G. C. T.) and longitude.
3. Make usual altitude corrections.
4. N. A. for declination.
5. Solve by 211 method for *t*.
6. Combine *t* with longitude for G. H. A.
7. N. A. to find the G. C. T. of this G. H. A.
8. Difference of this G. C. T. from chronometer equals part or all of error.
9. If body was sun, moon, or planet, go back to #4 and use the revised G. C. T. found in #7 to pick out more exact declination. Use it in repeating #5 and repeat remaining steps through #7. Difference of this second revised G. C. T. from chronometer will show practically all error.

NOTE: The method for #7 of finding from N. A. what the G. C. T. is for a certain G. H. A. is the same as the method of finding G. C. T. of transit. (See Part III, Chap. 30, Problems, Transit.)

13. Azimuth and Compass Error

AZIMUTH of a celestial body is the angle at the zenith between the meridian of observer and the vertical great circle passing through zenith and the body. It is usually measured from the north in north latitudes, east or west through 180°, and similarly from the south in south latitudes and designated Z. It is also measured from the north point clockwise through 360° and is then identical with true bearing and labeled Z_n. (*See* FIG. 37.)

Azimuth of a body is needed for two main purposes: (1) for drawing a line of position, as will be explained in the next chapter, and (2) to determine the error of a magnetic compass.

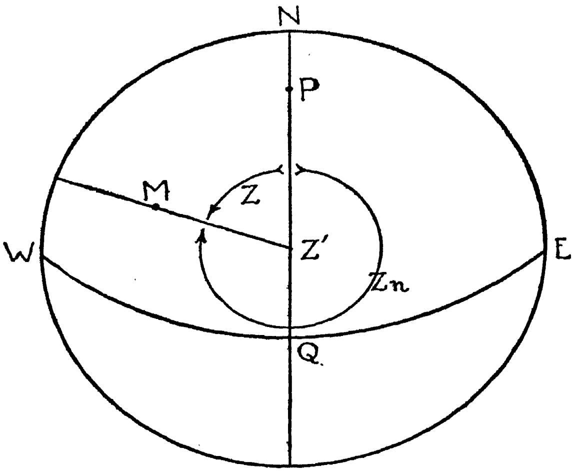
Azimuth is found by the following means:

1. Gyro compass (with azimuth circle).
2. Table.
3. Formula (with and without sextant observation).
4. Diagram (with and without sextant observation).
5. Celestial Co-ordinator.
6. N. A. (after obtaining latitude by Polaris).

The first is possible because of the fact that gyros show true directions with no deviation or variation.

The second is much used. H. O. 71 ("Red" Azimuth Tables for declinations 0° – 23° and to Lat. 70°, usually for the sun) and H. O. 120 ("Blue" Azimuth Tables for

declinations 24° — 70° and to Lat. 70°, for stars) are the standard books. Entering with *L, d,* and *t* (the first two in even degrees and the last in 10-minute time intervals) gives *Z*. A rather long and tedious calculation is necessary in most instances because the quantities usually lie between



Projection of the Celestial Sphere on the Plane of the Horizon.

WQE = Equator
Z′ = Zenith
P = Elevated Pole
M = Body
Z = Azimuth direct from Elevated Pole
Z_n = Azimuth by 360° system

FIG. 37. Azimuth.

these entering values and interpolation is necessary. "Cugle's Two-Minute Azimuths" recently published in two volumes are more expensive, but also save some time. They are arranged on the same basis as the H. O. volumes except for having 2-minute time intervals. They cover lati-

tudes to 65° and declinations to 23°. H. O. 66, Arctic Azimuth Tables, is a small volume, also on the same basis, for latitudes 70° – 88° and declinations to 23°. It is limited to the hours 4 to 7 A. M. and 5 to 8 P. M. at 10-minute intervals. H. O. 200 includes a short table which is entered with h_o, d and t and requires some interpolation. It covers declinations to 89° 30′.

The new series of volumes, H. O. 214 (*see* Chap. 16), provides probably the most satisfactory means of similarly finding azimuth. L is in whole degrees, d in whole and usually half degrees and t in whole degrees which corresponds to 4-minute time intervals. The volumes already published carry latitudes to 79° and declinations to 74° 30′. (A problem showing method of interpolation will be found in Chapter 30.)

There are many formulas for azimuth, but they all fall into the following three classes:

Time Azimuth (Z obtained from t, p, and L). The equations and method are lengthy and will not be given here but may be found in Bowditch.

Altitude Azimuth (Z obtained from h, p, and L). Collins' equation (U. S. N. I. P. July 1934) is recommended:

$$\text{hav } Z = \sec h \sec L \sin (s - h) \sin (s - L)$$

where $s = \frac{1}{2}(h + L + p)$

Hinkel (U. S. N. I. P. June 1936) gives the following for use with H. O. 211:

$$\text{Sec}^2 \tfrac{1}{2} Z = \frac{\sec s \sec (s - p)}{\sec h \sec L}$$

In either case it is measured from N. in N. lat. and from

S. in S. lat. The quantity $s - p$ may have a negative value but this does not matter since the secant of a negative angle less than 90° is positive. (*See* Table 14.)

Time and Altitude Azimuth (Z obtained from t, h, and d). The Bowditch equation is:

$$\sin Z = \sin t \sec h \cos d$$

There is a defect in this method in that nothing indicates whether the azimuth is measured from north or south. However, as the approximate azimuth is always known, the solution will almost always be evident. When in doubt, with sun almost E. or W. and L and d same name, find altitude when sun *is* E. or W. as follows: $\sin h = \csc L \sin d$. If observed altitude was less, sun was on side toward elevated pole, etc.

Using H. O. 211:

$$\csc Z = \frac{\csc t \sec d}{\sec h}$$

An azimuth diagram is often accurate enough for practical purposes and saves much of the time ordinarily spent in calculating formulas or interpolating tables. A copy of Rust's azimuth diagram comes with one of the short-cut systems, Weems' "Line of Position Book." Entering with t, h, and d one arrives at Z. Another diagram by Capt. Weir is mentioned by Dutton. Weir's uses t, d, and L to obtain Z.

A Celestial Co-ordinator (*see* Chap. 7) set for L, t and d will give approximate values for Z and h.

The azimuth of Polaris may be found from the *Nautical Almanac* as follows:

1. Find approximate latitude by procedure given in Chapter 11 under Latitude by Polaris.

2. Enter Table IV (p. 285 in 1943 N. A.) with L. H. A. and approximate latitude and take out azimuth.

Summary

Time Azimuth

1. G. C. T. and L. and Lo. (or t).
2. N. A. for G. H. A. and d ($p = 90° \pm d$).
3. G. H. A. and Lo. for t (unless given).
4. Table (H. O. 66, 71, 120, 214 or Cugle's) or formula or diagram (Weir).

Altitude Azimuth

1. Sextant altitude.
2. G. C. T. of observation, and L.
3. Make usual altitude corrections.
4. N. A. for d ($p = 90° \pm d$).
5. Formula.

Time and Altitude Azimuth

1. Sextant altitude.
2. G. C. T. of observation, and Lo.
3. Make usual altitude corrections.
4. N. A. for G. H. A. and d.
5. G. H. A. and Lo. for t.
6. Formula, or Table (H. O. 200), or diagram (Rust).

Azimuth of Polaris

1. Latitude by Polaris.
2. N. A. Table IV.

Compass Error

The compass azimuth, taken with an azimuth circle, of a body, usually the sun and preferably at low altitude, is compared with the true azimuth of the body, which is determined by one of the methods described. The difference is the error: variation plus deviation. The difference between this total error and the variation, obtained from the chart, is the deviation on the particular heading.

14. Sumner Lines of Position

CAPT. THOMAS H. SUMNER, an American shipmaster, on December 18, 1837, near the end of his ship's voyage from Charleston, S. C., to Greenock, Scotland, was in need of data as to his position. About 10 A. M. an altitude of the sun was obtained and chronometer time noted. As the D. R. latitude was unreliable, two additional latitudes 10′ and 20′ farther north were assumed and the three possible longitudes were worked out. When the three positions were plotted on the chart, they were found to be in a straight line. "It then at once appeared that the observed altitude must have happened at all the three points . . . and at the ship at the same instant." The conclusion was that, although the absolute position of the ship was uncertain, she was necessarily somewhere on that line. Capt. Sumner published his discovery in 1843 and it is considered, with the previous invention of the chronometer and the subsequent development of the Saint-Hilaire method, as one of the three greatest contributions to the science of navigation.

Circles of Equal Altitude

If we look up at the top of a vertical flagpole from a certain distance away from its base, we will be gazing upward at a certain angle. We may walk all around the pole keeping at the same distance, but this angle will not change.

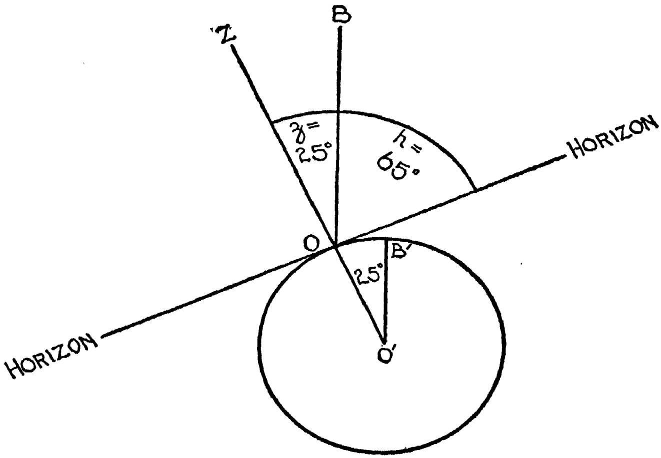
It is obvious that a heavenly body directly over the north pole would show the same altitude from every point on a given parallel of north latitude.

It is somewhat less obvious but none the less true that, if a certain heavenly body is in the zenith at a point *anywhere* on the earth at a given time, and if circles in the same hemisphere are imagined around this point as a center, then the altitude of such body from every point of any one circle at the given time will be the same. Measuring the radius of such a circle in degrees on the earth's surface, the largest possible circle, dividing the earth into hemispheres, would have a radius of 90°. From any point on this circle of 90° radius, the body would be in the horizon and have an altitude of 0°. From the center with radius 0°, the body would be in the zenith and have an altitude of 90°. On a circle with radius of 10° the body's altitude would be 80°, etc. The radius is, therefore, the complement of the altitude and so equals the zenith distance. Expressing it in minutes of arc, the zenith distance equals the number of nautical miles the observer is away from the center of the circle, where the body is in the zenith. (*See* FIG. 38.)

In an astronomical sight, the following is learned about *the point on the earth in whose zenith the body is:*

1. Longitude—from G. C. T. and G. H. A.
2. Latitude—from d.
3. Distance from observer—from h.

There is nothing to show at what point on the circle of equal altitude the observation was made. But an observation of another body preferably in a direction at right angles



★ = Extremely distant fixed star
Circle = Earth in plane of vertical circle of body observed
Z = Zenith of observer
OB = Line from observer toward body
h = Altitude
z = Zenith distance
B′ = Place on earth where body is in zenith (geographical position)
ZO is prolonged to center of earth at *O′*
O′B′ is parallel to *OB* considering distance of body

Therefore, by geometry: Angle *OO′B′* = angle *ZOB*, or angular distance on earth from observer to body's geographical position = zenith distance of body at place of observer (for nearer bodies, parallax correction makes them equal anyway).

FIG. 38. Zenith distance and the Radius of the Circle of Equal Altitude.

to the first (or the same body several hours later) can be made in order to get a new circle.

This will intersect the first circle at two places, one of them being the ship's position. But as the ship's position is always approximately known within about 30 miles, and as these two intersections may be thousands of miles apart, there is no question as to which is the correct one.

The Line of Position

It is never necessary to determine the whole of a circle of equal altitude. A very small portion of it is sufficient, and such an arc may be considered as a straight line for the length needed to cover the probable limits of the position of the observer. Such a line is known as a Sumner line or line of position. It gives a knowledge of all the probable positions, while a sight worked with a single assumed latitude or longitude gives only one probable position. It always lies at right angles to the direction of the body from the observer, as a tangent to a circle through a point is perpendicular to the radius at that point.

In the two earlier line of position methods, sights were worked for latitude when the body was nearer north or south and for longitude when the body was nearer east or west. The three methods of determining a line of position are as follows:

1. *Chord method:* for one sight assume two values of latitude and determine longitudes or assume two values of longitude and determine latitudes. Two points are thus fixed on the chart and the line joining them is the line of position.

2. *Tangent method:* for one sight assume one latitude or one longitude and determine the other co-ordinate. One point on the line is thus obtained. The azimuth of the body must now be found either by formula, table, or diagram, and a line drawn from the one point at this angle in the direction of the body. Then perpendicular to this line and through the point is drawn the line of position. When using the time-sight formula for longitude, it is convenient to also use the altitude azimuth formula. When using the Phi Prime, Phi Second formula for latitude, one should use the time and altitude formula for azimuth.

3. *Saint-Hilaire method:* assume both latitude and longitude using either the D. R. position or one nearby and calculate what the altitude and azimuth of a body would be, there, at the time an actual altitude was taken on the ship. The difference between the actual and the calculated altitude shows how far to move along the azimuth line from the assumed position before drawing a perpendicular. This is the line of position. (This method will be discussed in more detail in the next chapter.)

15. Saint-Hilaire Method

THE FRENCHMAN Adolphe Laurent Anatole Marcq de Blond de Saint-Hilaire was born at Crécy-sur-Serre (Aisne) July 29, 1832 and entered the French Navy in 1847. He was made a Commander of the Legion of Honor in 1881 and reached the rank of Contre-Amiral in 1884, following distinguished service in the Tunis expedition. He died in Paris, December 30, 1889. Little information is available about him. His method of the calculated altitude was published as "Calcul du point observé. Méthode des hauteurs estimées," in *"Revue Maritime et Coloniale,"* Vol. XLVI, pp. 341-376, August 1875. He split the astronomical triangle by dropping a perpendicular from the body onto the meridian and solved the resulting right spherical triangles by logarithms, using the D. R. position.

Lord Kelvin before the Royal Society, February 6, 1871, had announced a method of comparing calculated with actual altitude for locating the line of position. He used an assumed position and computed tables for solving the triangle. These were published as "Tables for Facilitating Sumner's Method at Sea" in 1876.

It would be interesting to know if Saint-Hilaire knew of Kelvin's announcement, and when Saint-Hilaire first developed his method.

Everything so far in this Primer has been leading up to the Saint-Hilaire method. It is a most important development in the science of position finding and is the basis of practically all modern navigational systems.

Details of Procedure

1. Take an altitude and note G. C. T.
2. Make usual altitude corrections.
3. Note D. R. latitude and D. R. longitude.
4. N. A. for G. H. A. and d.
5. Combine G. H. A. and D. R. longitude for t (for D. R. position).
6. With t, d, and L using one of several formulas to be given and perhaps tables or a diagram, calculate altitude (h_c) and azimuth (Z). These are the values the body would have had if the observation had been made at exactly the D. R. position.
7. The difference between your observed altitude corrected as in #2 and the calculated altitude of #6, in minutes of arc, represents the difference in miles between *your actual distance* from the place where the body is in the zenith, called its geographical position, and the *distance of the D. R. position* from the body's geographical position. (We could compare the observed zenith distance with a calculated zenith distance but the altitude way is more convenient.) This difference is called the altitude difference or intercept and is given the abbreviation a. If the observed altitude is greater than the calculated, it means your actual position was nearer the body's geographical position than was the D. R. position. (You were more "under" the body.) But if your observed altitude is less than the calculated, then you were really farther from the body's geographical position than was the D. R. position. (You were less "under" the body.) The a is, therefore, labeled "toward" or "away."

8. Through the D. R. position on the chart draw a line at the calculated azimuth angle in the direction of the body's geographical position.

9. Lay off on this line from the D. R. position a distance in miles equal to the intercept, either *toward* or *away* from the body's geographical position, as the case may be.

10. Through the end of this intercept draw a line perpendicular to the azimuth line. This is the line of position. (*See* FIG. 39.)

One of the best formulas for calculating the altitude called for in #6 is known as the cosine-haversine and is described by Dutton as "the most widely used formula of trigonometry applied to nautical astronomy. It is universally applicable to all combinations of the values of *t*, *d*, and *L*." Here it is:

$$\text{hav } z = \text{hav } (L \sim d) + \text{hav } \theta$$

where $\text{hav } \theta = \text{have } t \cos L \cos d$

from which $h_c = 90° - z$

(L and d: add opposite names, subtract likes)

Convenient to use with this is the time and altitude azimuth formula:

$$\sin Z = \sin t \cos d \sec h_c$$

Other formulas or tables or a diagram may also be used.

The cosine-haversine formula and a formula for azimuth should be memorized once and for all by every student navigator. Knowing these one could be independent of all short-cut systems and could function even when only tables of a foreign publication were available.

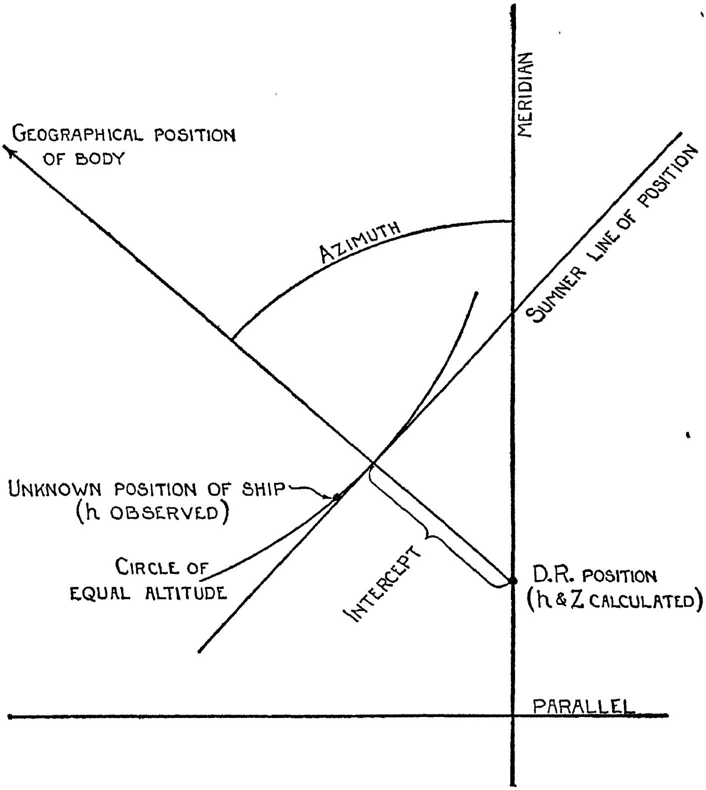


FIG. 39. The Saint-Hilaire Method.

The Fix

We have seen that one line of position does not fix the exact location of the ship. Another line must be obtained which will intersect the first giving what is called a *fix,* and so show the real position. The second line preferably should be at about 90° to the first although a smaller angle down to 45° may be effective. The ideal fix is probably from three stars at about 120° intervals around the observer, giving 3 lines which should intersect at a point or at least with formation of a small triangle. When the same or another body is used later in the day to get the second line, the first must be moved forward parallel to itself according to the course and distance made good. These latter factors are estimates from compass and log readings or engine revolutions and open to error through current, wind, etc. Hence a line which is over five hours old in a moving ship is not trusted. A fix obtained after moving forward a previous line is known as a *running fix.*

The best method for a series of two or more star sights is to use the D. R. position at the time of the last of the series in working each sight. Then plot the line for the last sight in the usual way but advance the lines of the previous sights according to time difference, speed and course of ship to get fix at time of last sight.

When chart work is impossible, a method for computing the position of the intersection of position lines (Bowditch, pp. 190-3) may be followed. This requires knowledge of the position of one point on each line and so the lines had best be obtained by the tangent method. (*See* Chap. 14.)

The various practicable combinations of observations for a fix are as follows:

Daylight
Separated
Sun and Sun
Simultaneous or close
Sun and Moon
Sun and Venus
Venus and Moon
Twilight
Simultaneous or close
Star and Star
Star and Planet
Star and Moon
Planet and Planet
Planet and Moon

The Computed Point

This is the point where a line from the D. R. position dropped perpendicular to the line of position intersects the latter. It is the mean of the possible positions of the ship on the line. It, therefore, is the best point to use as the ship's probable position in the absence of an intersecting line and without contrary evidence from weather or current.

16. Short-Cut Systems

A RESUMÉ OF NAVIGATION METHODS in tabular form by Soule and Collins, copyrighted by the U. S. Naval Institute, was published by the U. S. Hydrographic Office as "Supplement to the Pilot Chart of the North Atlantic Ocean" in 1934. It gives a ready comparison of the salient features of twenty-nine different methods for determining the elements of the astronomical triangle in order to plot the line of position. There are seventeen systems working from an assumed position and three from the D. R. position, making twenty for the Saint-Hilaire method. The remaining nine are longitude methods for chord or tangent lines. Since this publication, I know of five more methods which have appeared: Aquino's "Tangent Secant Tables," "Hughes' Tables for Sea and Air Navigation," Ageton's "Manual of Celestial Navigation" (for D. R. or A. P.) and H. O. 214 (all of which use the Saint-Hilaire method) and Aquino's method for finding latitude and longitude directly.

Assumed *vs.* D. R. Position

The advantages of using an assumed position are that fractions of degrees of latitude or hour angle may be avoided and arithmetic simplified. Dutton (7th ed., p. 204) states that any position correct to within about 40′ may be used. "No matter what position is assumed within the 40′ limit, the line of position will plot in the same place on the

chart, the altitude difference, *a*, changing with each change of assumed position." The chief disadvantage is that chart work is complicated by using a different assumed position for each of several sights taken in a short interval. This means drawing each azimuth line through a different point. Another bad feature is that long altitude intercepts may result.

The advantages of working from the D. R. position are: it is run forward on the chart from the last fix; only the one position is plotted for any group of sights; errors are more apparent; much smaller altitude intercepts result which makes for accuracy; and (theoretically) the azimuth should be more accurate. The disadvantages are that a little more arithmetic and time are required.

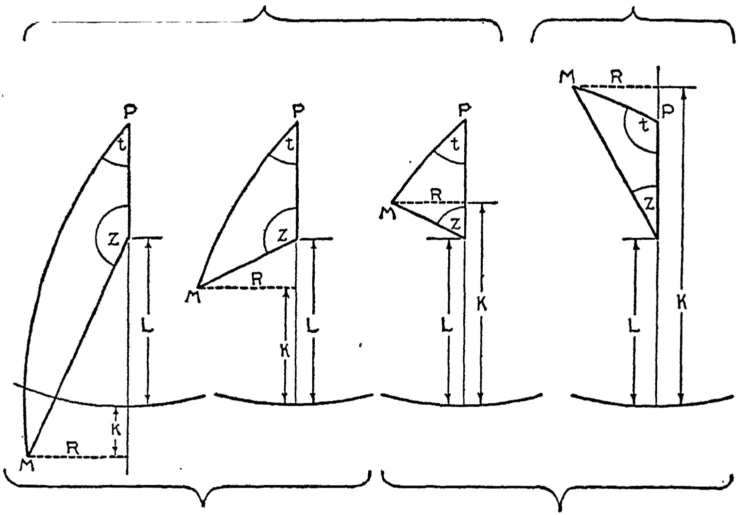
In view of the foregoing and after using both, the writer has chosen the D. R. position system. Of the various methods and tables available, he recommends the use of "H. O. 211" by Comdr. A. A. Ageton, U. S. N. Its full title is "Dead Reckoning Altitude and Azimuth Table."

H. O. 211

Advantages of H. O. 211 are as follows: It gives sufficient accuracy without interpolation; except for H. O. 214, it is the shortest D. R. method; it follows a uniform procedure for sun, moon, star or planet; it consists of only one table of 36 pages running in two columns of almost all whole numbers; the table is indexed and very simple and easy for rapid use; azimuth is determined without question as to origin; and there are only two special rules. (*See* FIG. 34.)

The only disadvantage is that when the hour angle lies

Rule 1: K is always $< 90°$. Take K from top of table, except when t is $> 90°$. Take K from bottom of table.



Rule 2: Z is always $> 90°$. Take Z from bottom of table, except when K is same name as $\&>$ latitude. Take Z from top of table.

Of course, if $K = L$ then $Z = 90°$; if $t = 90°$ then $K = 90°$.

K takes same name as declination (N. or S.)

FIG. 40. Rules for H. O. 211.
(Projection of celestial sphere on plane of horizon. R is dropped from body M, perpendicular to meridian.)

within about 3° of 90° the solution requires such close interpolation that Ageton recommends discarding those sights.

The table consists of double columns of log cosecants and log secants each $\times 10^5$ which eliminates decimal points in all but a few pages. Without interpolating between the values, which are given for each half-minute of arc, accuracy (excluding, of course, sextant errors) is within ½ mile.

The formulas used, derived from Napier's second rule, are as follows:

$$\csc R = \csc t \sec d$$

$$\csc K = \frac{\csc d}{\sec R}$$

$$\csc h_c = \sec R \sec (K \sim L)$$

$$\csc Z = \frac{\csc R}{\sec h_c}$$

Give K same name as d.
$K \sim L$: subtract likes, add opposites.

NOTE: See Part III, Chap. 29, for convenient forms to use with H. O. 211. See Ageton, A. A.: "The Secant-Cosecant Method for Determining the Altitude and Azimuth of a Heavenly Body." *U. S. Naval Institute Proceedings.* Oct. 1931, Vol. 57, No. 344, pp. 1375-1385.

H. O. 214

This recent development in navigational tables is entitled "Tables of Computed Altitude and Azimuth." The complete set will consist of nine volumes. Eight have already

been published covering latitudes to 79° (10° to a volume). Either the D. R. or an assumed position may be used. The procedure in either case is the shortest of its kind possible at present. The azimuth from the table may be interpolated for the D. R. position if extreme accuracy is desired. Tables for star identification are included at the end of the data for each degree of latitude. The fact that the whole set is of nine volumes need not discourage the average cruising yachtsman, since each volume covers 600 miles in latitude. A few of the volumes will probably be sufficient in most cases.

In Chapter 30 will be found a fix from two stars worked out by H. O. 214 with both the D. R. position and the assumed position methods.

It will be seen that the method which uses the D. R. position for each sight exceeds in length the method using a different assumed position for each by two short altitude corrections (L and t) and a combination of the three corrections, for each sight.

However, the method using a different assumed position for each sight requires the calculation of the assumed longitude, sometimes by a subtraction, and the plotting of each azimuth line from a different point. The intercepts are usually much longer from assumed positions and the azimuths not quite so accurate.

The following summaries will show each step.

H. O. 214: Method Using D. R. Position

1. Take an altitude by sextant and note G. C. T.
2. Make usual altitude corrections for h_o.

3. Note D. R. Latitude and D. R. Longitude.

4. Find d and G. H. A. in N. A. and t from G. H. A. & D. R. *Lo.*

5. Enter tables with nearest whole degree of latitude and local hour angle and nearest whole or half degree of declination.

6. Copy the 4 values found which will be h, Z, $\triangle d$ and $\triangle t$. These last two represent the change in altitude due to a change of 1′ of arc of declination and of local hour angle.

7. Calculate by inspection the differences between figures used to enter tables and actual values of d, t (D. R.) and L (*D. R.*).

8. Use table on back cover pages to multiply d and t differences (found in #7) by $\triangle d$ and $\triangle t$ respectively. Give each result a + or a − sign according as the tables show the altitude to be increasing or decreasing in passing from the chosen tabulated value toward the value to be used. These are altitude corrections for d and t.

9. Use another table in back of book entering with L difference (found in #7) and azimuth (found in #6) and take out altitude correction for L. *With* $Z > 90°$*:* if D. R. latitude exceeds chosen tabulated value, give L correction a − sign; otherwise mark it +. *With* $Z < 90°$*:* if D. R. latitude exceeds chosen tabulated value, give this L correction a + sign; otherwise mark it −.

10. Combine these three altitude corrections algebraically.

11. Combine the resulting total with altitude from table (found in #6) to get h_c.

12. Combine h_c and h_o for intercept.

13. Plot from D. R. position.

H. O. 214: Method Using Assumed Position

1. Take an altitude by sextant and note G. C. T.
2. Make usual altitude corrections for h_o.
3. Note D. R. Latitude and D. R. Longitude.
4. Find d and G. H. A. in N. A.
5. Assume the whole degree Latitude nearest D. R.
6. Assume Longitude nearest D. R. which when combined with G. H. A. will give a whole degree for t.
7. Enter tables with assumed latitude and local hour angle and nearest whole or half degree of declination.
8. Copy the values found for h, Z and $\Delta\ d$. This last represents the change in altitude due to a change of 1′ of arc of declination.
9. Calculate by inspection the difference between declination used to enter tables and the actual declination.
10. Use table in back cover pages to multiply d difference (found in #9) by $\Delta\ d$. Give result a + or − sign according as the tables show the altitude to be increasing or decreasing in passing from the chosen tabulated value toward the value to be used. This is the altitude correction for d.
11. Combine this correction with altitude from table (found in #8) to get h_c.
12. Combine h_c and h_o for intercept.
13. Plot from the assumed position.

17. Special Fixes

APART FROM THE USUAL TYPE of fix described in Chapter 15, there are several methods for certain circumstances, some of which will now be described.

The Zenith Fix with Sumner Arcs

When the altitude of a body is over 89° its zenith distance is, of course, a matter of less than 60′. It will be recalled that the zenith distance of a body in minutes of arc represents the distance in nautical miles from observer's position to the geographical position of the body, namely, that point on earth which has the body in its zenith. (*See* FIG. 38.) Ordinary lines of position under these circumstances are not accurate because a straight line here will only coincide with a very small part of the circle of equal altitude. It is better to draw the actual circle or an arc of it. This can be done as follows:

Take a sextant altitude 3 or 4 minutes before transit, note G. C. T. and make usual corrections. The G. H. A. of the body at this G. C. T., obtained from N. A., represents the west longitude of its geographical position. The declination represents the latitude of its geographical position. This point can, therefore, be plotted on the chart or plotting sheet. With the point as a center and with dividers set by the latitude scale for the number of miles equal to minutes of zenith distance, an arc can be swept in the general direction of the ship. Following a similar observation from the

same position about 3 or 4 minutes after transit, another arc from a new center can be similarly swept which will intersect the first and give a fix. In case the ship has meanwhile moved, the first arc can be advanced by shifting its center according to course and distance covered to give a fix at second observation. Or the second arc can be pushed back similarly to give a fix at first observation. For accuracy, a third arc from an observation at transit may be used, but is not essential.

The azimuth in this situation is rapidly changing from east of the meridian to west of it. If body is to transit between zenith and elevated pole, azimuth (on the 180° system) will diminish to 0° and then increase. If body is to transit with zenith between it and elevated pole, azimuth will increase to 180° and then diminish.

Theoretically, this method should apply to any body passing near zenith but it is unlikely that a star would be found and satisfactorily observed in this way at twilight. With the usual method available of a fix from two stars there would be no need for it.

Practically, it is of occasional use in daylight with the sun at noon or Venus at transit. These bodies will only pass near zenith if the ship is in low latitudes and when declination and latitude are about equal. (*See* Lecky, 22nd ed., p. 500; Dutton, 6th ed., p. 212; Dugger, U. S. N. I. P. Jan. 1936.)

The Noon Fix with Equal Altitudes

This is described by Lecky (22nd ed., pp. 482-4). Noon latitude is obtained by a meridian altitude in the usual way. If the ship is practically stationary and the sun's altitude

not less than 75° the longitude may be found as follows. Use another sextant. From 10 to 15 minutes before local apparent noon observe sun's altitude and note G. C. T. Clamp the sextant. Observe sun with this clamped sextant after noon until sun is seen on horizon as before and again note G. C. T. The half-sum of these two times will be G. C. T. of apparent noon at ship. G. H. A. obtained from N. A. for this G. C. T. will equal longitude.

The Fix on Equator at Equinox

On the two equinox dates of each year the sun rises practically due east and sets practically due west throughout the earth except in extreme polar regions. At the equator it will pass through zenith at noon, the azimuth remaining 90° all day from either pole, E. in A. M. and W. in P. M., except for the instant when it is 0° at noon. Set the sextant at 90° − × (*see Note*) and observe sun eastward in A. M. till its lower limb just meets horizon and note G. C. T. (A quick look with sextant to N. or S. should show no change if you are actually on equator.) G. H. A. obtained from N. A. for this G. C. T. will equal long.; lat. = 0°.

NOTE: × represents the corrections for height of eye and semidiameter which must be allowed for in order to observe the sun at a true altitude of 90°. (Parallax and refraction do not operate when body is in zenith.) If the sextant were set for 90°, the subsequent application of these corrections would show a true altitude in excess of 90° when observation is made from the usual height of a ship's bridge.

For example:

Table A.	Corr. for sun (S. D.) alt. 90° =	+ 16′
"	B. Corr. for sun (S. D.) Mar. 21 =	+ 0′.1
"	C. Corr. for 39 feet (H. E.) =	− 6′.1
	Total	+ 10′.

Reversing the sign and applying to 90°

$$\begin{array}{rr} 90^\circ & 00' \\ & -10' \\ \hline 89^\circ & 50' \end{array} = \text{sextant setting.}$$

At any other time of day on equator at equinox, if a sextant altitude is taken, and corrected, then $t = 90° - h_o$ and t combined with G. H. A. for time of observation = longitude; latitude = 0°.

The Fix on Equator Not at Equinox

Hinkel (U. S. N. I. P. June 1936) gives the following equation for local hour angle, to be worked by H. O. 211, when on equator, not at equinox, but with a definite value for the sun's declination.

$$\sec t = \frac{\csc h}{\sec d}$$

Combining t with G. H. A. gives longitude; latitude = 0°.

Aquino's Fix from Altitude and Azimuth

Capt. Radler de Aquino of the Brazilian Navy in his article "A Fix from Altitude and Azimuth at Sea and in the Air" (U. S. N. I. P. Dec. 1936) presents a method of obtaining a fix from simultaneous sextant and compass observations of a single body. The azimuth must be taken with a gyro compass in perfect order or with a magnetic compass whose error is definitely known. The student is referred to the original article for details, equations used

and their derivation but it is sufficient to state here that latitude and longitude are obtained directly from the reasonably short calculation. A form for the procedure is given in Chapter 29 and a sample problem in Chapter 30. Any tables containing log tan, log sec and log csc for every 1′ of arc may be used. Aquino has prepared an especially convenient table of these functions which is contained in his "A Navegaçao Hodierna com Logaritmos de 1633!" This can be purchased from R. de Aquino, 133 Rua Raul Pompeia, Copacabana, Rio de Janeiro, Brazil. The tables occupy only 18 pages. The practical difficulty in this otherwise ideal method is, of course, in obtaining a sufficiently accurate azimuth.

The Fix with Weems' Star Altitude Curves

Star Altitude Curves, published by the Weems system of Navigation, Annapolis, Maryland, represent a new development in position finding and under favorable conditions offer the means for obtaining a fix in the amazingly short time of two minutes.

The following items are dispensed with: assumed position, *Nautical Almanac,* right ascension, hour angle, azimuth and the plotting of position lines.

The student is referred to the publisher for details or can find a good description in Dutton, 7th ed., pp. 352-5.

Very briefly, each page is a "grid" formed by the respective equal altitude curves, or lines of position, of three stars plotted on a Mercator chart at 10′ intervals against latitude (left and right edges) and local sidereal time (top and bottom). One of the stars is usually Polaris.

At the top are shown the names of the stars used on that page and the half or whole hour of local sidereal time which is covered. Each page covers about 10° of latitude and the collection covers latitudes from 0° to 70° 30′ and local sidereal time from 0^h to 24^h.

No correction is needed when the altitudes are observed with a bubble sextant. With a standard sextant, only the correction for dip of the horizon is used.

For any place and sidereal time, a circle of equal altitude of a star remains nearly the same from year to year. An annual correction is indicated under the name of each star at the top.

A second-setting watch on G. S. T. is used, or else G. C. T. is converted to G. S. T.

The procedure in simplest form, for northern hemisphere, is as follows:

1. Apply approximate longitude (in time) to G. S. T. to obtain approximate L. S. T.
2. Turn to the corresponding page for this time and for the approximate latitude.
3. Observe altitude of one of the "longitude stars" and note G. S. T.
4. Observe altitude of Polaris. (Time of this is not necessary, as altitude of Polaris changes so slowly.)
5. Find the proper altitude line and fraction on the Curves for the longitude star and do the same for Polaris. Mark the intersection of these two lines.
6. Project this intersection horizontally to the scale to find latitude.
7. Project this same point vertically to the scale to find L. S. T.

8. Take the difference between this and G. S. T. of observation to find longitude in time.

9. Convert above to arc to find longitude.

This remarkable procedure is at present most suitable for high-flying fast airplanes. The limitation to the three stars given on each page without curves for sun, moon or planets, make it a supplement to, rather than a substitute for, the usual methods of marine celestial navigation.

18. Polar Position Finding

THE AMATEUR NAVIGATOR will rarely be concerned with lines of position in arctic regions. However, two reasons have led me to include something of these matters in this Primer: first, we will be completing the story of position finding for the whole earth instead of the usual limit of 65° latitude N. or S.; second, there is practically nothing printed about polar position finding in any of the texts or manuals in common use.

Let us consider for a moment certain special conditions which exist at, say, the exact geographical north pole. After understanding these, we can more easily understand the conditions in the *neighborhood* of the pole which are similar but which become less so, the farther away from the pole we go, until they gradually become those with which we are already familiar in latitudes below 66°.

At the North Pole

At the north pole, then, every direction radiating out from an observer is south.

Circles drawn with observer as center will be leading west clockwise or east counterclockwise.

The horizon of an observer at the north pole marks the equinoctial, or celestial equator, above which all bodies are in northern declination. A north declination as given in N. A. will then be the true altitude. Sextant altitudes,

of course, would differ by the amount of the usual corrections.

The sun at Spring equinox, March 21, appears above the horizon and skims around it in about 24 hours. It remains visible and rises gradually to an altitude of about 23° 27′ at summer solstice, June 21. It then gradually sinks to disappear at Fall equinox, Sept. 23, and remains invisible for the six months of arctic night.

The moon's declination is north about one half of each month and at such times, of course, this body will be above the horizon. It sometimes attains an altitude of over 21°. The presence of the sun does not make it invisible and so when the two are in evidence they can often be used for lines of position which intersect and give a fix.

All stars of northern declination are theoretically visible during the arctic night from Fall to Spring equinox. As their declinations change so little from year to year, this upper half of the "inner surface of the celestial sphere" presents an unvarying picture except for the moon and planets.

The four navigational planets appear at times during arctic night. Naturally their different orbits and varying declinations will change the program from year to year. In 1938, for instance, Venus was in northern declination from March 16 to August 9, Mars from February 1 to October 29, Jupiter at no time, and Saturn from March 6 to the end of the year but never as much as 5°, which is too low an altitude for navigational use. Conditions favoring visibility of a planet in northern declination during the six months of arctic daylight would be: more altitude, more angular distance from the sun, and less bright sunlight.

There is no azimuth at the pole, since true north is in the zenith.

Polaris when visible will be found within 1° 1′.6 of the zenith.

To find direction of Greenwich: choose a heavenly body, find its G. H. A. in N. A. for a given instant, set a pelorus with this G. H. A. on the body at the time chosen, and 0° on the pelorus will point toward Greenwich. The direction of any other place will be its longitude west, read in degrees on pelorus, as 74° for New York City.

The north point of the magnetic compass will point south toward the magnetic north pole or approximately along the meridian of longitude 97° W. (The latitude of the north magnetic pole is about 70° N.)

A gyro compass, started with N. point toward Greenwich, for instance, theoretically should hold its direction steady in relation to the universe and so appear to make one clockwise revolution in one sidereal day, as the earth turns under it counterclockwise. "Actually, however, this result would not be mechanically possible due to friction of the supports. This would produce a slight tilt of the gyro axis, which in turn would result in a continuous precession about the vertical axis due to pendulousness of the compass." (Sperry Gyroscope Co., Inc. communication.)

Owing to ice floes making the horizon rough, a bubble sextant is necessary. This supplies its own horizon.

Being on top of the spinning world, the rule that "day and night are equal at equinox" is not quite true at or near the pole. One has to get away far enough to be able to have some of the earth come between oneself and the sun to make any night possible at all.

In Polar Regions

The behavior of the magnetic compass in polar regions deserves some mention. At the north magnetic pole it becomes useless inasmuch as the directional force is to pull the north end downward. A dipping needle supported on a horizontal axis proves this. Getting away from the magnetic pole the compass gains in directional force, but this never becomes as strong as it is in our latitudes. Suppose the compass is carried around the geographical north pole on the parallel of latitude 80° N. Starting north of the magnetic north pole, the north point of the compass will point south and variation = 180°. Continuing east on the 80th parallel, this variation will be west and will steadily decrease till a point is reached about halfway around the parallel when compass will be pointing true north and variation = 0°. Still continuing east, the compass will begin to show east variation which will steadily increase till our starting point has been reached when variation again = 180°.

The nature of the Mercator chart makes it useless for high latitudes and so a polar great circle chart is used. (*See* Chap. 21.) A straight line drawn on this represents a portion of a great circle. This is usually sufficiently close for a true line of position although the latter is always part of a "small circle." (A straight line on a Mercator chart similarly is a rhumb line and not truly a part of a small circle.) A method for allowing for curvature in long position lines will be explained.

The astronomical triangles in polar problems are peculiar in that the side between elevated pole and zenith is

TABLE 17

FOR CORRECTING POLAR POSITION LINES

True Alt.	100′		200′		300′		400′		500′		550′		600′		650′		700′	
	Z	H	Z	H	Z	H	Z	H	Z	H	Z	H	Z	H	Z	H	Z	H
10	.3	.3	.6	.7	.9	2.1	1.2	3.8	1.5	6.2	1.6	7.5	1.8	9.1	1.9	10.6	2.1	12.7
15	.5	.4	.9	1.5	1.4	3.5	1.8	6.2	2.2	9.6	2.5	11.7	2.7	13.2	2.9	16.2	3.1	18.9
20	.6	.5	1.2	2.1	1.8	4.8	2.3	8.6	3.0	13.3	3.3	15.8	3.6	18.9	3.9	22.2	4.2	26.2
25	.7	.7	1.6	2.7	2.3	6.2	3.2	11.0	3.9	16.9	4.3	20.5	4.6	24.2	5.0	28.5	5.4	32.7
30	1.0	.8	1.9	3.5	2.9	7.8	3.8	13.5	4.8	21.0	5.3	25.3	5.7	30.1	6.2	35.9	6.7	41.4
35	1.2	.9	2.3	4.0	3.5	8.8	4.7	16.1	5.8	25.3	6.4	30.4	6.9	36.1	7.5	42.3	8.1	49.0

relatively much shorter. However, this does not prevent, except in extreme cases, using H. O. 211 for solutions or the cosine-haversine formula for calculated altitude. H. O. 66 "Arctic Azimuth Tables" may be used in certain limited conditions. (*See* Chap. 13.) Weems' "Line of Position Book" Polar edition 1928, is a system using an assumed position for which the tables have been extended to include all latitudes.

Weems, in an article "Polar Celestial Navigation" (U. S. N. I. P. Nov. 1933), suggested a method preferred for position finding within 5° or 10° of the pole. Briefly it is as follows:

Note D. R. position on polar chart, Greenwich meridian up.

Assume observer is at pole, in which case $h_c = d$ and G. H. A. is what corresponds to Z, measured clockwise for N. pole, counterclockwise for S. pole.

Take sextant altitude of sun, note time and make usual corrections $= h_o$.

Find G. H. A. and d from N. A. for this time.

Plot line from pole toward sun's geographical position at the proper G. H. A. and extend this line through pole.

Find difference in minutes of arc between d (representing h_c) and h_o. If latter is greater, then position line is toward sun from the pole and vice versa. Call this difference a.

Lay off a from pole on the line drawn through pole, and draw through its outer extremity a preliminary position line perpendicular to sun G. H. A. line.

Measure distance on this position line between sun's

G. H. A. line and observer's approximate position on the position line.

Use Table 17, from Weems' article, entering with the above distance and the observed altitude to get corrections for h_o and Z. These are needed because the position line really curves if the above distance is considerable. The correction for Z will be added or subtracted according to obvious circumstances.

Plot new sun bearing line from observer's approximate position on first position line using corrected Z, measuring from a zero line parallel to Greenwich meridian.

Lay off from starting point toward sun on this new sun bearing line the correction for h_o obtained above, and draw through its extremity a perpendicular to the new sun bearing line. This perpendicular will be the corrected line of position.

Repeat this whole process for moon when it bears about 90° from sun.

Intersection of sun's and moon's corrected lines = fix.

Any other pair of available bodies at suitable angles may be used.

19. Identification

AT NIGHT, after the horizon has faded, with a clear sky it is easy to identify most of the bright stars because various groups are easily recognized and relationships to such groups are obvious.

At evening twilight, however, before the horizon has blurred, there will often be only a very few stars visible and these widely separated with no groups to give clues of identity. One may have only a few minutes to take altitudes of such stars before the horizon becomes useless. Identification may be necessary and even a vital matter in a case where an observation is long overdue and perhaps only one star is seen for a moment between clouds. The approximate bearing of the star should be noted as well as the sextant altitude and G. C. T. If the latitude is fairly well known, identification is made as follows:

1. H. O. 127 is first used. With latitude, altitude and azimuth (from N. or S. according to latitude) as arguments, take out declination and hour angle (local, in time).
2. Convert L. H. A. from time to arc.
3. Combine L. H. A. with longitude to get G. H. A. of star at G. C. T. of observation.
4. You must now reduce this to the G. H. A. at 0^h same date in order later to find the star in the N. A. So find the table in N. A. entitled "Correction to be Added to Tabulated Greenwich Hour Angle of Stars." Use the G. C. T. in this table and take out the corresponding angle.
5. Subtract this angle from G. H. A. at observation (add-

TABULAR SUMMARY OF METHODS *Table 18*

GIVEN	OBTAIN	PROCEDURE	RESULT
Sextant, corrected Chronometer→GCT & NA→GHA & DR Lo & GHA DR	h_o d t L	H. O. 211 method or H. O. 9 Cosine Haversine	Altitude, calculated, for comparison with observed, in St. Hilaire method for Line of Position
As above As above Calculation as above	d t h_c	H. O. 211 method, or Time & Alt. Az. formula, or H. O. 200, Az. table, or Az. diagram (Rust)	Azimuth, calculated, for use with above, in St. Hilaire method for Line of Position
Sextant, corrected GCT & NA→GHA & Assumed or DR Lo & GHA Assumed or DR	h_o d t L	H. O. 214 tables	Altitude & Azimuth, calculated for comparison with observed altitude, in St. Hilaire method for Line of Position
Azimuth Circle Compass GCT & NA→GHA & Known or DR Lo & GHA DR or observation	Z C d t L	H. O. 66, 71, 120, or 214, or Cugle's Tables, or Az. diagram (Wier)	Azimuth, true, for comparison with observed to get total compass error. This combined with variation taken from chart gives Deviation of compass on course
Azimuth Circle Compass GCT & NA→GHA & d & 90°±d Known or DR Lo & GHA DR or observation	Z C p t L	Time Az. formula	
Azimuth Circle Compass Sextant, corrected GCT & NA→GHA & d & 90°±d DR or observation	Z C h_o p L	Altitude Az. formula (Collins) or H. O. 211 (Hinkel)	
Calculation for apparent sun noon or transit of other body Sextant, corrected, at above 90°−h NA for time of observation	GCT h_o z d	L & d opposite names: z−d L & d same names & L > d: z+d L & d same names & L < d: d−z L & d same names (lower transit): 180°−(d+z)	Latitude from meridian altitude
Calculation for apparent sun noon or transit of other body Sextant within 28 minutes of above, corrected NA for time of observation DR	GCT h_o d L	H. O. 9 Table 29 (DR L & d)→a H. O. 9 Table 30 (a & time from transit)→at² h_o+at² or h_o−at² (lower transit)= Meridian altitude. Follow previous procedure.	Latitude by reduction to meridian of ship at obs.
Sextant, corrected GCT & NA→GHA & Known Lo & GHA	h_o d t	Phi Prime, Phi Second formula or H. O. 211 (Favill) Body < 3h, from meridian " < 45° " " in Z " > 3° in d	Latitude
Sextant, corrected GCT & NA→GHA & d & 90°±d DR or observation	h_o p L	Time Sight formula or H. O. 211 (Hinkel) Body near E or W	t (& GHA)→ Longitude
Sextant, corrected Approx GCT & NA→d & 90°±d DR or observation Known	h_o p L Lo		t (& Lo)→GHA (& NA)→GCT of obs. This compared with approximate GCT of obs. gives Chronometer Error
Sextant, corrected (star or planet) GCT Azimuth Circle DR or observation DR or observation	h_o Z L Lo	H. O. 127 or 214	d & t (& Lo)→GHA at GCT of obs. NA→angle for this GCT. Subtracting it from GHA of obs.= GHA at 0^h GCT. Find body in NA having this GHA and d for Identification
Sextant, corrected (star) Special watch Sextant, corrected (Polaris)	h GST h_o	Star Altitude Curves (Weems)	Latitude LST (& GST)→Lo in Time (& to Arc)= Longitude
Sextant, corrected Gyro compass GCT & NA→GHA &	h_o Z d	Aquino's formula	Latitude Longitude

ing 360° to G. H. A., if necessary) and get G. H. A. at 0^h.

6. Look in N. A. star tables of the month of the observation and under the proper date find star which has approximately this G. H. A., and the declination already found in H. O. 127. This will be the star observed.

In case no star is found whose G. H. A. and declination approximate those you have determined, a search should be made through the data of the four navigational planets and one will probably be found which will meet requirements and so settle the identification.

In the rare instance where the body is neither one of the 54 usual stars nor one of the 4 navigational planets, it will probably prove to be one of the 110 additional stars whose mean places are given in two pages of N. A. Only R. A. and *d* without G. H. A. values are provided for these and so the older method of sidereal time must be employed. Use H. O. 127 as above to get *d* and L. H. A. (in time). Finding G. S. T. by the usual calculation and then applying D. R. longitude (in time) to get L. S. T., the latter is combined with the hour angle taken out of H. O. 127 to get the star's R. A. Using this R. A. and the *d* obtained from H. O. 127, search the list of 110 additional stars and make the identification. (NOTE: This and the fix by Weems' star altitude curves are the only procedures given in this Primer where the use of sidereal time is essential.)

If the H. O. 214 identification tables are used instead of H. O. 127, the local hour angle will be obtained in arc direct. *See* the N. A. for practical suggestions on identification.

20. Tabular Summary

TABLE EIGHTEEN has been prepared to give at a glance the outlines of fourteen types of calculation which have been under consideration. Several others might have been included but it is felt that this list is sufficient for our purposes. Even after months of study the student will find it hard to definitely remember the exact steps of a process unless he is using it constantly. This table, it is hoped, will serve as a refresher and will surely save time that would be otherwise wasted in repeatedly looking back through the text. The arrangement in four columns is self-explanatory and horizontally traces a calculation from start to finish under the headings of Given, Obtain, Procedure, and Result.

See Table 18 inserted.

Part III: Supplementary

21. Sailings; D. R.; Current

CELESTIAL NAVIGATION, strictly speaking, does not include the subject of sailings, dead reckoning, or current. However, the methods we have recommended for position finding at sea all require a D. R. position to start from and so it is proper to include something on the means by which such a D. R. position is ascertained.

We saw in Chapter 10 that the D. R. position was obtained by keeping track of courses steered and distances run from the last well known position. We will see presently how such data is translated into a new latitude and longitude.

We also saw in Chapter 10 that a rhumb line was a curved line on the earth's surface which intersected all meridians at the same angle.

The term *sailings* is used for various means of solving problems involving calculation of a ship's position in relation to a place left or to a place one is approaching. The following items come into consideration:

Course: the constant angle a rhumb line makes with meridians.

Distance: the length of the rhumb line in nautical miles between starting point and destination. This usually refers to a day's run more or less and not to a whole voyage.

Difference of latitude: the number of degrees, etc., the latitude is altered by the run made. Expressed in minutes it equals nautical miles.

Difference of longitude: the number of degrees, etc., the longitude is altered by the run made.

Departure: the number of nautical miles of easting or westing made good by the run. It is also the length in nautical miles of an arc of a parallel. (Do not confuse these uses of the word departure with its use in the expression *"take a departure"* which consists in fixing the position of the ship, before losing sight of land, by some landmarks and using this position as the starting point for dead reckoning.)

The matter of departure requires still further comment. For a small area of the earth's surface, where its spherical form may be neglected, the departure will be the same, whether measured on the parallel of the point left or on that of the point reached. (*See* FIG. 41.)

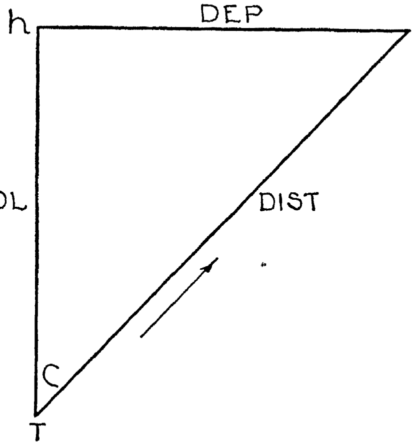
The formula Dep. = Dist. sin C may be shown (*see* Dutton) to hold for any rhumb line and any distance. As so computed for a rhumb line, no matter how long, Dep. is *not* the easting or westing measured on the parallel of the point left or on that of the point arrived at. It is the distance steamed east or west made up of the sum of the Deps. of any number of small triangles each constructed like those in Figure 42. This sum proves to be *practically* equal to the Dep. between the places measured on a parallel in the middle latitude of the two places.

The Sailings

I. *Plane* (earth's surface assumed to be flat) considers: Course, Distance, Diff. Lat., Departure.

Types: Single, Traverse.

Solved by: Logs, Traverse Tables, Construction on plotting sheet or chart.



C = Course
$Dist$ = Distance
DL = Difference of latitude
Dep = Departure

T = Starting point
T' = Destination
h = Intersection of line from T' dropped perpendicular to meridian of T

By definition:

$$\text{Sin } C = \frac{\text{Dep}}{\text{Dist}}$$

$$\text{Cos } C = \frac{\text{DL}}{\text{Dist}}$$

$$\text{Tan } C = \frac{\text{Dep}}{\text{DL}}$$

Therefore:

$$\begin{cases} \text{Dep} = \text{Dist sin } C = \dfrac{\text{Dist}}{\text{csc } C} \\ \text{Dist} = \dfrac{\text{Dep}}{\text{sin } C} = \text{Dep csc } C \end{cases}$$

$$\begin{cases} \text{DL} = \text{Dist cos } C = \dfrac{\text{Dist}}{\text{sec } C} \\ \text{Dist} = \dfrac{\text{DL}}{\text{cos } C} = \text{DL sec } C \end{cases}$$

$$\begin{cases} \text{Dep} = \text{DL tan } C = \dfrac{\text{DL}}{\text{cot } C} \\ \text{DL} = \dfrac{\text{Dep}}{\text{tan } C} = \text{Dep cot } C \end{cases}$$

FIG. 41. Plane Sailing.

II. *Spherical* (earth's true shape used) considers: Course, Distance, Diff. Lat., Departure, Diff. Long.

Types: Parallel, Middle Latitude, Mercator, Great Circle, Composite.

Solved by: Methods similar to those for Plane.

Plane Sailings

Plane sailings can be solved by plane trigonometry and logarithms as indicated in Figure 41, using Bowditch Table 32 for logs of numbers and Table 33 for logs of functions of angles. It will be seen that there are two or more formulas for the solution of each side of the triangle to provide for the various combinations of data given and to be found. The two commonest problems are: first, given course and distance, to find difference of latitude and departure; second, given difference of latitude and departure, to find course and distance. In the latter problem, the formulas are not suitable for long distances.

A shorter way is to use the traverse tables. Bowditch Table 3 gives Diff. Lat. and Dep. for each unit of distance from 1-600 under each degree of Course.

The simplest way, and one sufficiently accurate for yachtsmen, consists in laying down the various given terms by scale upon a position plotting sheet and then measuring the required terms.

A *Single* calculation is all that is usually called for.

The *Traverse* problem arises when a ship has made an irregular track sailing on several different courses. It consists in finding the difference of latitude and departure for each course and distance by the traverse tables and reducing

all to a single equivalent course and distance. This is done by tabulating the amount of miles made good to north or south and to east or west on each course; adding up each of the four direction columns; taking the algebraic sum of the north and south totals for Diff. Lat. and of the east and west totals for Dep.; and finding in the traverse table the course and distance corresponding to this Diff. Lat. and Dep. This resulting course and distance can then be drawn on the plotting sheet from the starting point and will show the D. R. position at the end of the irregular track.

Spherical Sailings

Parallel sailing problems come up when a ship's course is due east or west. There is no change in latitude and all the distance covered is departure. The problem is to find the Diff. Long. corresponding to the Dep. This may be done by logs using the formula (*see* Dutton, Chap. 1):

$$\text{Diff. Long.} = \text{Dep. sec } L$$

Or, if it is desired to find the amount of departure necessary to reach a certain longitude:

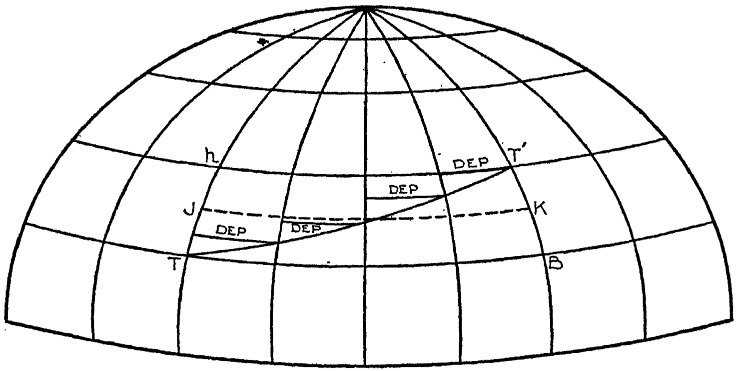
$$\text{Dep.} = \text{Diff. Long. cos } L$$

It may be more convenient to use Bowditch Table 3. Diff. Long. will be found in minutes of arc or should be converted thereto as the case may be. The labels of the table should now be imagined changed as follows:

Course	$\longrightarrow$	Lat.
Dist.	$\longrightarrow$	Diff. Long.
Lat.	$\longrightarrow$	Dep.

(*See* Bowditch for explanation of how this is possible.) Pick out the two known values and read off the one desired.

Middle latitude sailing is founded on the assumption that the departure between two places with different lati-



Departure between T & $T' = JK$

(Proof depends on calculus and the difference in latitude should not be great and latitudes must not exceed 50°.)

FIG. 42. Middle Latitude Sailing.

tudes equals the length in miles of that parallel of latitude which lies midway between the parallels of the places and between the meridians of the places. This is sufficiently accurate for a day's run under latitude 50°. (*See* FIG. 42.)

Given latitude and longitude of starting point, course, and distance made, to find latitude and longitude arrived at.

Bowditch Table 3 is used entering with course and distance to find Diff. Lat. and Dep. as in Plane Sailing. Con-

verting minutes to degrees, etc., this Diff. Lat. is applied to latitude left to give *latitude arrived at.*

The two lats. are now averaged to find Mid. Lat.

The problem thus becomes one in parallel sailing and Table 3 is used with the same changes of labels as described for parallel sailing, as follows:

Entering with Mid. Lat. (Course) and Dep. (Lat.) we find Diff. Long. (Dist.). Convert minutes to degrees, etc. Applying to longitude left gives *longitude arrived at.*

Given latitude and longitude of both starting point and point reached, to find course and distance made good.

Subtract lesser latitude from greater to get Diff. Lat. and convert to minutes.

Average the two latitudes to find Mid. Lat.

Combine the two longitudes to find Diff. Long and convert to minutes.

Entering Table 3 with Mid. Lat. (Course) and Diff. Long. (Dist.) we find Dep. (Lat.).

Entering Table 3 with Diff. Lat. (Lat.) and Dep. we find *course and distance.*

When the places are on opposite sides of the equator, this method is not applicable.

Mercator sailing is used when the distance involved is greater than can be accurately handled with middle latitude.

As previously described, a Mercator chart has parallel meridians. The result is that the farther one goes from the equator, the more unnaturally extended are the parallels of latitude between any two meridians. Proportions are kept right because the meridians between parallels of latitude become more and more extended. Expressing it in another way:

Globe map:

Long. degrees become shorter toward pole.
Lat. degrees stay same.

Mercator chart:

Long. degrees stay same toward pole.
Lat. degrees become longer.

The length of the meridian as thus increased on a Mercator chart between equator and any given latitude, expressed in minutes as measured on equator, constitutes the number of *Meridional Parts* corresponding to that latitude. Bowditch Table 5 gives meridional parts or increased latitudes for every minute of latitude between 0° and 80°.

Given course and distance from a known position we wish to know latitude and longitude arrived at. This is accomplished as follows:

1. Use Table 3 entering with course and distance and find figure in Lat. column, which is difference of latitude expressed in nautical miles or minutes of arc. (Or use logs and equation Diff. Lat. = Dist. cos C.)

2. Reduce above to degrees and minutes.

3. Apply above to latitude left (adding if course was northerly, subtracting if southerly) and obtain *latitude arrived at.*

4. When both the place left and place reached are on the same side of the equator, look up in Table 5 the meridional parts for each latitude and subtract the smaller quantity from the greater. When the places are on opposite sides of the equator, similarly look up the two quantities but add them. In each case the result is the *meridional difference of latitude* or *m*.

5. Table 3 is now used with the following two substitutions of title:

$$\begin{array}{lcl} \text{Lat.} & \longrightarrow & m. \\ \text{Dep.} & \longrightarrow & \text{Diff. Long.} \end{array}$$

Enter table with course and m (Lat. column) and take out Difference of Longitude (Dep. column) expressed in minutes. (Or use logs and equation: Diff. Long. $= m \tan C$, if C is not near E. or W.)

6. Reduce above to degrees and minutes.
7. Apply above to longitude left (adding if course was away from Greenwich, subtracting if toward) and obtain *longitude arrived at.* (If over 180°, subtract from 360° and change name.)

In case we require course and distance by rhumb line between two given positions which are far apart:

1. By subtraction find difference of latitude.
2. By subtraction find difference of longitude.
3. Use Table 5 and find m as described in 4 above.
4. Find *course* by using logs and equation:

$$\tan C = \frac{\text{Diff. Long.}}{m}$$

5. Find *distance* by using logs and equation:

$$\text{Dist.} = \text{Diff. Lat. sec. } C.$$

Great Circle Sailing

The shortest distance over the earth's surface between two places is that measured along the great circle which passes through them. Generally speaking, the economy of

the great circle over the rhumb line is greatest for long distances at high latitudes when the course lies more E. or W. than N. or S.

Every great circle, excluding the equator and meridians, cuts successive meridians at a different angle. To keep a ship on a great circle course, therefore, would require constant change of heading. As this is impracticable, the course is changed at regular intervals such as every 150 or 300 miles. Thus the ship really follows a series of rhumb lines.

Every great circle track, if extended around the earth, will lie half in the northern and half in the southern hemisphere.

The *Vertex* of a great circle track in one such hemisphere is that point which is farthest from the equator, or has the highest latitude.

A great circle track between two places in, say, the northern hemisphere will everywhere be north of the rhumb line between these places. This is because a rhumb line is a spiral, concave toward the pole, which approaches but never reaches the pole. On a Mercator chart, then, a great circle track between two places in the northern hemisphere appears as a curve above the straight rhumb line joining the places.

Great Circle or *Gnomonic Charts* are charts on which great circle courses will appear as straight lines. They are constructed for a given area of the globe by passing a plane tangent to the center of the area and then projecting by rays from the globe's center all features of the area onto the plane. As the plane of every great circle passes through earth's center, and as one plane always intersects another in a straight line, the projected great circles will be straight lines.

The *Polar Chart* is a great circle chart for use in high latitudes where a Mercator is too distorted and spread out. The meridians are straight lines radiating from the pole and the parallels are circles with the pole as center, increasingly separated from the pole outward. (*See* Dutton, Chap. VI.)

Hydrographic Office Publications 1280-1284 are great circle charts of the principal oceans. They are spoken of as on the *Gnomonic Projection.* The meridians appear as straight lines converging toward the poles, and the parallels appear as non-parallel curves, concave toward the poles. No compass rose can be applied to the whole of such a chart and latitude and longitude at a particular point must be determined by reference to nearby meridians and parallels. The chart becomes distorted in the longitudes more removed from the center and so is not satisfactory for navigation. However, it provides the most convenient means for determining great circle track and distance between points. The track is always a straight line. An explanation is printed on such charts of how to determine the length of the track and the course at any point of the track. After selecting a number of points of the track on the great circle chart, their latitude and longitude are determined and such points are then plotted on a Mercator chart. A fair curve is then drawn through these points and gives the great circle on the Mercator.

In the absence of a great circle chart, the track, courses, and distance may be determined by computation. This rather long process will not be gone into here. A good explanation and method will be found in H. O. 211. A very easy and rapid method of finding initial course and distance is provided for in H. O. 214.

Composite Sailing

This is a combination of great circle and parallel sailing. It is used when the great circle course between two points passes through higher latitudes than it is thought wise to enter. This may be from considerations of ice or cold or wind. There are three main ways of determining the track; by gnomonic charts, by computation, or by graphic methods. (*See* Bowditch.) An approximation to the shortest track between the points without exceeding the given latitudes is had by following a great circle between the points until the limiting parallel is reached, following this parallel until the great circle is again met, and finally following the great circle to destination.

The track may be laid on a Mercator chart as follows:

1. Draw the track on a great circle chart.
2. Determine the latitude and longitude of a number of points of the track.
3. Transfer these points to the Mercator.
4. Draw a smooth curve between them from point of departure to destination.
5. Discard the portion of the curve which lies north (or south) of the limiting parallel, retaining the two remaining portions of the great circle and that part of the limiting parallel included between their northern (or southern) ends.

Other methods of composite sailing are the following:

1. One great circle to a predetermined place; a Mercator to another predetermined place; and another great circle to destination.
2. One great circle to the limiting parallel at a given longitude; another great circle from there to destination.

3. One great circle from departure tangent to the limiting parallel; another great circle from destination tangent to the limiting parallel; course along parallel between the two points of tangency. This is the shortest possible composite course.

W. S. N. Plotting Charts

A new series of charts or plotting sheets is featured by the Weems System of Navigation. The charts are designed to eliminate the sailings calculations. Mercators and great circles are measured directly. There are four charts in the series covering all areas from equator to pole.

Dead Reckoning

The working of dead reckoning involves a combination of the methods of traverse and middle latitude sailing. *See* Chap. 29 for appropriate form of table to be kept and the following summary from Bowditch:

"When the position of the vessel at any moment is required, add up all the differences of latitude and departure, and write in the column of the greater, the difference between the northing and southing, and the easting and westing. Apply the difference of latitude to the latitude of the last determined position, which will give the latitude by D. R., and from which may be found the middle latitude; with the middle latitude find the difference of longitude corresponding to the departure; apply this to the longitude of last position, and the result will be the longitude by D. R."

Dutton says, p. 95: "In modern practice nearly all navigators do their dead reckoning work graphically."

Current

Current is a broad term covering anything and everything that causes a discrepancy between the D. R. position for a given instant and a fix from celestial observation for the same instant. Dutton summarizes as follows:

1. Foul bottom of ship.
2. Unusual condition of trim.
3. Error of patent log or revolution curve.
4. Inaccurately known compass error.
5. Poor steering.
6. State of wind and sea.
7. Observation errors.
8. Real ocean currents or streams.
9. Tidal currents found along the coast.

The Estimated Position

Either a Computed Point (Chap. 15) or a D. R. position which has been revised for Current is called an Estimated Position.

Real Ocean Currents

The *Set* of a current is the direction toward which it is flowing.

The *Drift* of a current is its velocity in knots (nautical miles per hour).

The two usual problems which arise may be solved by traverse tables or trigonometry but simple graphic solutions on the chart with a protractor are satisfactory, as shown (after Dutton) on pages 176-179, following.

Current does not affect the ship's speed through the water. Wind, weather, waves, etc., however, do so.

In case the current is directly against or directly with the ship's course, the algebraic sum of speed and current gives speed over ocean floor.

$$\text{Time required} = \frac{\text{Distance in miles}}{\text{Speed over bottom in miles per hour}}$$

$$\text{Speed over bottom in miles per hour} = \frac{\text{Distance in miles}}{\text{Time consumed}}$$

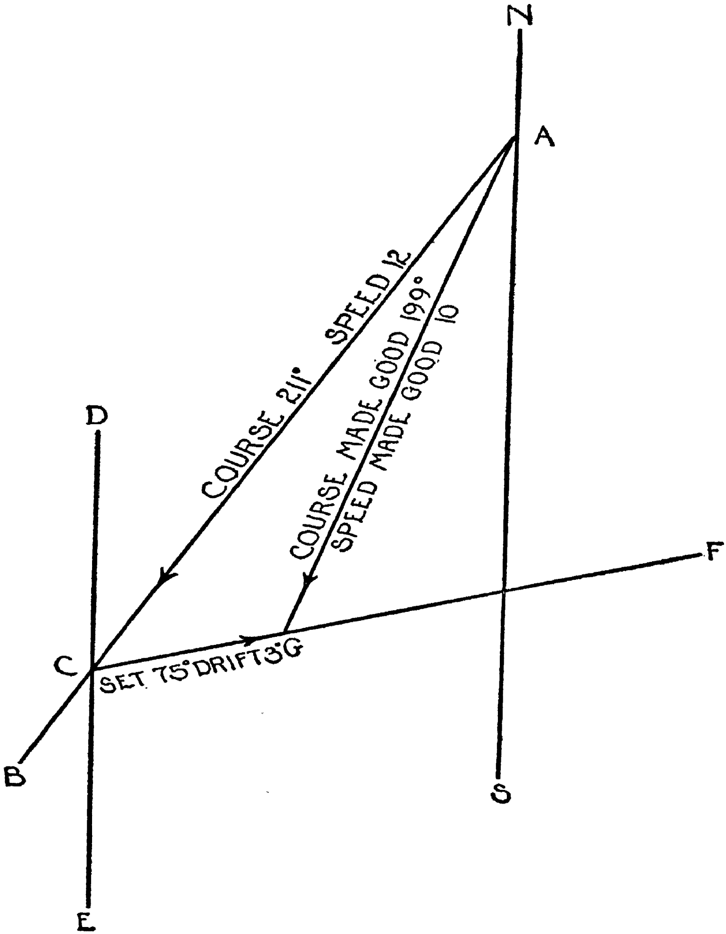


FIG. 43. First Current Problem.

First Problem

Given:

Course and speed through water.

Set and drift.

To find:

Course and speed over ocean floor.

Example: (*See* FIG. 43.)

Ship was at A.

Steamed on course 211° at 12 knots.

Through current of set 75° and drift 3 knots.

To find course and speed over ocean floor:

Solution:

Draw $N\ S$ meridian through A.

Lay off clockwise angle $N\ A\ B = 211°$, for course.

Using scale of chart make $A\ C = 12$ miles, for speed.

Draw $D\ E$ through C parallel to $N\ S$.

Lay off clockwise angle $D\ C\ F = 75°$ for set.

Lay off $C\ G = 3$ miles by same scale, for drift.

Draw $A\ G$.

Then clockwise angle $N\ A\ G =$ course made good over ocean floor $= 199°$.

And $A\ G$ by same scale $= 10$ miles $=$ speed of 10 knots over ocean floor.

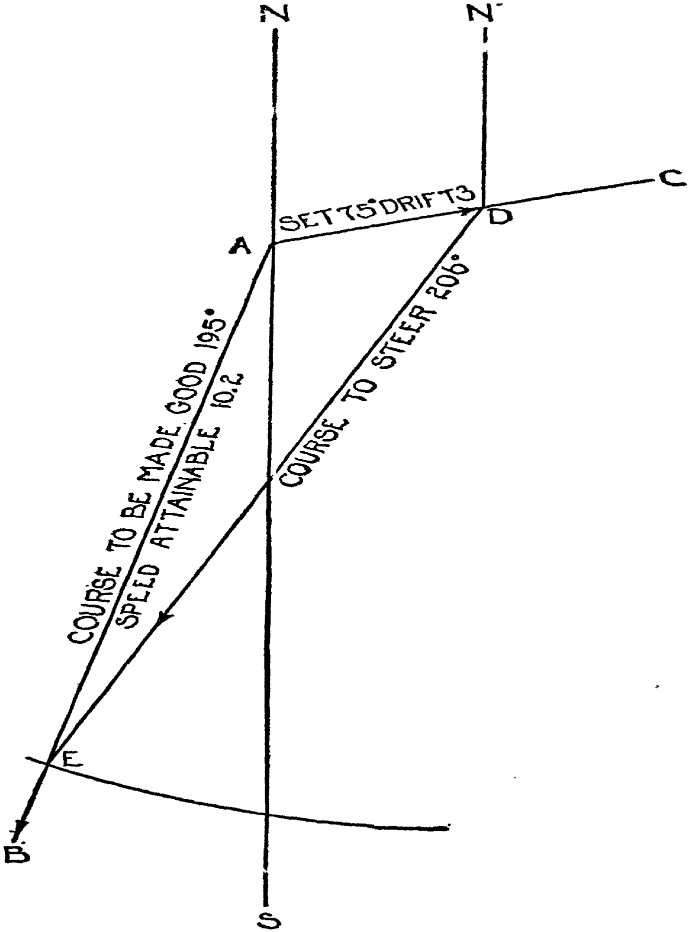


FIG. 44. Second Current Problem.

Second Problem

Given:

Set and drift.
Speed of ship
Course you desire to make good.

To find:

Course you must steer.
Speed you will make on it.

Example: (*See* FIG. 44.)

Ship is at A.
Current has set 75° and drift 3 knots.
Ship's speed = 12 knots.
Course to be made good = 195°.
To find course to steer and speed attainable on it:

Solution:

Draw $N\ S$ meridian through A.
Lay off clockwise angle $N\ A\ B$ = 195°, for course to be made good.
Lay off clockwise angle $N\ A\ C$ = 75°, for set.
Using scale of chart make $A\ D$ = 3 miles, for drift.
Erect a meridian $D\ N'$.
With D as a center and a radius of 12 miles by same scale, sweep an arc cutting $A\ B$ at E.
Draw $D\ E$.
Then clockwise angle $N'\ D\ E$ = course to steer = 206°.
And $A\ E$ by same scale = 10.2 miles = 10.2 knots speed attainable on course to be made good.

22. The Day's Work

BEGINNING A VOYAGE, a good departure should be taken while landmarks are still in view. This becomes the start of the dead reckoning which with new departures from fixes by celestial navigation is kept up till the voyage is completed. The patent log, having been put over, is read and recorded on taking departure. If an engine revolution counter is used it is read and time noted. The following summarizes the required daily work at sea (*see* Dutton):

Dead Reckoning is carried forward from one well-determined position to the time of next observation or fix. Comparison of D. R. and fix gives "current" since last fix. Log or engine revolution counter is read at each observation.

Compass Error is determined at morning and afternoon observations of the sun and, if possible, whenever the course is changed.

Sun Observations should be made in forenoon and afternoon when on the prime vertical providing the altitude is then over 10°; otherwise, as near prime vertical as possible with altitude exceeding 10°. Sun should also be observed at local apparent noon or for ex-meridian.

Fixes from at least two stars should be made at morning and evening twilight. Other combinations as star and planet, planet and moon, etc., can be used.

Additional work, in case of failure to obtain A. M. fix or other of the above observations, consists in daylight sights

of sun and moon, or sun and Venus, for a fix; latitude by Polaris may be done at dawn; or a running fix from two sun sights may be all that can be obtained.

Chronometers are wound at a certain time each day and note made of any changes between them.

Radio time signals are received daily, if possible, and chronometer correction noted.

Reports of work are made to the captain at 8 A. M., noon, and 8 P. M. These include:

Position by D. R.

Position by observation.

Set and drift of current.

Deviation of compass.

Course and distance made good since last report.

Course and distance to destination.

Any necessary change in course.

Ship's run (at noon since last noon).

23. Essential Equipment

(For Celestial Navigation and Dead Reckoning)

Absolute minimum

Sextant
Chronometer
Watch
Magnetic Compass
Azimuth Circle or Pelorus
Patent Log
Nautical Almanac
Dead Reckoning Altitude and Azimuth Table (H. O. 211)
Bowditch (H. O. 9)
Plotting sheets and charts
Dividers, parallel rulers, triangle, magnifying glass
Notebook, pencils, eraser, pencil sharpener

Also important

Radio
Gyro Compass
Engine revolution counter
Telescope or binoculars
Stop-watch
Second-setting 24 h. face watch
Course protractor
Tables of Computed Altitude and Azimuth (H. O. 214)
Noon-Interval Tables (H. O. 202)
Star Identifier (H. O. 2102-B)
Celestial Globe (preferred to above)
Collected Correction Tables (*see* Chap. 3)

24. Practical Points

General

ON BEGINNING a voyage, the navigator must provide himself with the latest ocean charts of the regions to be sailed, and note the date of the last corrections. All weekly "Notices to Mariners" should be obtained which have appeared subsequent to this date and any applicable data therein should be entered on the charts. The material necessary for coastwise navigation and piloting such as tide and current tables, coast pilots, light lists and large scale charts of coasts, harbors, etc., are not part of celestial navigation and are mentioned here only in passing. The essential equipment for taking and working sights should be checked over carefully and put in order.

The navigator's watch (pocket or wrist) should be kept on zone time for ease of translation to G. C. T.

The best standard of accuracy attainable in position finding may be expressed as a circle with a radius of one mile around the fix point on the chart. This is practically perfect work. It may occasionally be bettered, but is often not attained. Refraction, faulty observation, rolling ship, errors in time, etc., all make the actual practice at times more of an approximation than an exact procedure. Experience brings knowledge as to how accurate a given fix will probably prove to be.

The height of eye for the place on the ship which will usually be used by the navigator in taking sights should be accurately determined after the ship is loaded.

Sextant (General)

The minimum altitude for reliable observations is 10° and 15° is still better.

On a rolling vessel, stand on the center line to avoid changes in height of eye.

When a distant haze makes the horizon poor, it may be found better by lowering the height of eye.

In clear weather, increasing the height of eye makes for greater accuracy.

Index error of the sextant should be checked each time it is used.

"Bringing a body down to the horizon" is especially convenient for a star or planet sight but may also be well used for sun or moon. Set sextant at zero. Look through it directly at body. Loosen index arm and slowly push it forward. A second image of body will appear to drop from the original. Lower the aim of sextant, abandoning original image and following this second one which is in the mirror side of the horizon glass. Continue pushing index arm forward and lowering aim to keep the mirror image of body in view until horizon appears in clear half of glass. Clamp sextant at an approximate contact and adjust to exact contact by vernier screw.

After one preliminary observation, a sextant may be set ahead of a body's altitude on even 5′ or 10′ intervals of arc and several times of contact noted. Then use the average of these altitudes and times.

The author heartily endorses the following, written by J. T. Rowland in *Yachting,* March 1941: "I would rather use one good shot, in which I feel confidence, than introduce

the error which may be present in a doubtful one. My system, therefore, is to take a series of sights until I get one in which I have faith—and then use that. True this calls for some discernment on the part of the navigator but it is surprising how quickly one can develop an aptitude for 'calling one's shots.'

It is more important to be exact with time than with the altitude. Ten seconds of time error can throw a position off 2½ miles whereas 10″ of arc error will alter the result only about 1/5 mile.

When reading sextant, have the eye directly over the marks to avoid error of parallax between vernier and limb.

Focus the telescope on some distant object before screwing it in to sextant frame.

Eye-strain is less if both eyes are open when taking sights.

Between sights at twilight it may be helpful to close the eye used on sextant when going into the light to consult chart or read sextant. Returning to take another observation the sextant eye will then be in better shape for work.

Start with tangent screw about halfway from its two extreme positions so as to allow plenty of room for finer adjustment.

Sextant (Care of)

On small boats it is well to have a short lanyard attached, with an eye splice to fit over the wrist, in order to avoid dropping the sextant overboard.

The sextant is a delicate instrument and must not be dropped, bumped or jarred. It should be kept from sudden changes of temperature or moisture as far as is possible. It

should be in its case with index arm clamped when not working and the case should be attached to some safe place in the boat.

After use in wet weather the mirrors should be dried with chamois, linen or lens paper, not with silk.

When dirty, the arc and vernier may be cleaned with ammonia or sperm oil.

When the graduations of the sextant arc become dim, a paste of gritless lamp black and light oil may be smeared over arc and wiped off. This will clean the marks and also make them more easily read.

Sextant (Index Error)

Graduations of arc and vernier are continued somewhat to right of zero of arc and of zero (index) of vernier.

When a star's reflected image is brought into coincidence with the star seen directly, or when in daylight the reflected sea horizon is matched with the direct view of it, the sextant should read zero. If, however, zero of vernier lies to left of zero of arc (called "on"), the error is + and its correction will have to be subtracted. Or if zero of vernier lies to right of zero of arc (called "off"), the error is — and its correction will have to be added.

In the latter case, the reading of how much it is "off" is done in a special way. Read the vernier as usual, looking to left till a vernier line is found coinciding with an arc line, and count the minutes and seconds to this vernier line. Then subtract this amount from the maximum reading of the vernier. The result will be the amount of — error.

Another method of measuring index error is by measur-

ing the apparent diameter of the sun. This is done by first bringing the upper limb of reflected image tangent to lower limb of direct image and reading the sextant; then bringing lower limb of reflected image tangent to upper limb of direct image and again reading the sextant. Mark reading when on the arc as − and when off as +. The algebraic sum of the two readings will be *Index Correction.*

If a series of such observations is made, add up the readings of each pair without signs and divide by 2 to get sun's diameter. Look in N. A. for sun's semidiameter for the date and multiply by 2 for true diameter. Pick from the series that pair which most closely approximates this true value. Use this pair for Index Correction.

Artificial Horizon

One may use a pie plate of ink set in a box 18″ x 18″ x 8″ without a cover.

Sit back of and above plate in line with sun and its image in ink.

With sextant, bring real sun image down to ink image. Former will be in right-half of horizon mirror, latter in left. Though hard to find at first, lateral swinging will locate it.

In forenoon bring real sun image, lower limb, tangent to upper limb of ink image. Former will rise till transit, separating the images unless pulled down by tangent screw.

In afternoon, bring real sun image, lower limb, tangent to upper limb of ink image. Former will fall, causing overlap, unless raised by tangent screw.

Read the (double) sextant altitude of observation; apply I. C. to this; divide by 2.

Correct for refraction, parallax and semidiameter but not for height of eye.

Result is true altitude.

Sun

Swing in the strongest shade glass before observing sun. If this cuts off too much light use the medium shade glass, etc. If sun is looked at directly with too little protection it will make observation impossible for several minutes.

Test for index error after shade glasses are in place since they may be responsible for the error.

Hold sextant vertically, aimed toward the horizon beneath the sun. Move index arm from zero position slowly forward till sun's image appears in mirror of horizon glass and continue till lower edge of image is about tangent to horizon seen through clear half of horizon glass. Set the clamp screw (or release lever).

Determine vertical position of sextant by swinging it in an arc around line of sight axis and watching sun image swing in a curve convex to horizon. When image is lowest is when sextant is vertical. At this point adjust the tangent screw to bring sun's lower edge just tangent to horizon. At this instant call out "Mark," which signals your assistant to take the time. If alone, punch the starter of a stop-watch and proceed as described in Chapter II.

Owing to refraction when the sun's center is actually in horizon, the lower limb appears to be about one semidiameter above horizon.

The usual correction tables include correction for sun's semidiameter only when lower limb is used. If unusual

circumstances should permit only an observation of upper limb, as when sun is partly obscured by clouds or even in an eclipse, proceed as follows:

Apply I. C. to sextant altitude. Then subtract 2 × sun's semidiameter for closest date found in N. A. (This gives sextant altitude of lower limb.) Finally, apply usual corrections from Tables A, B, and C for refraction, parallax, semidiameter and dip, to obtain corrected altitude.

Moon

A bright moon may illuminate an otherwise dim horizon so that an altitude of the moon or a star nearby may be obtained.

At times with the moon about half-full, care must be taken to choose the limb whose upper or lower edge extends through what would be the vertical diameter of the moon if full.

Planets

Since navigational planets are brighter than stars they usually appear first at evening twilight and disappear last in the morning. Hence they can be observed when horizon is clearest and should be used when available.

When Venus is over 2 hours of R. A. distant from the sun, it may be seen in daylight. Calculate its approximate altitude and azimuth in advance. Set the sextant for this altitude and look through it with the medium telescope attachment in the direction of the calculated azimuth. Lenses and mirrors must be clean. A distinct white disc will

be found and exact altitude can then be obtained. Later, knowing where to look, Venus can be seen with the naked eye.

Stars

A way for timing star sights is as follows:

1. Start stop-watch at first observation.
2. Read stop-watch at 2nd. observation and record.
3. Read stop-watch at 3rd. observation and record.
4. Stop stop-watch on an even minute of chronometer.
5. Subtract total stop-watch reading from chronometer as in #4 for time of first observation.
6. Add stop-watch reading of #2 to above for time of second observation.
7. Add stop-watch reading of #3 to time of first observation for time of third observation.

Some use the following method for observing a star: Invert sextant set at zero, aim at star, push index arm forward till horizon is brought up approximately to star, clamp sextant, invert again and make final fine adjustment. This prevents starting to observe one star and ultimately making contact with another.

When observing a star, it is well to get its approximate bearing to aid in subsequent identification if doubt should exist.

The writer obtained a perfectly satisfactory altitude of Capella an hour before midnight 29 October, 1936, when a full moon and clear air made the horizon as distinct as could be desired.

Morning twilight is a better time for star sights, other things being equal, than evening. This is because the navigator may, by getting up a little earlier, pick out what stars he intends to use from the total display with no doubt of their identity, and then keep them in sight until the horizon has become sufficiently sharp for measuring their altitudes. Twilight is shortest in the tropics and increases with latitude.

When working two or three star sights it is best to put the data in parallel columns and work across the page. By this is meant figuring one item for each before going on to the next item, such as altitude, declination, hour angle, logs, etc. This is easier and errors show up more readily.

Avoid the use of the confusing inverting telescope. In poor light it is better to work without any telescope.

Latitude

In using sextant for meridian altitudes remember: Image goes up as body goes up.

Turning vernier screw down on left side increases altitude reading, and pulls body's image down.

Hence, while watching for body to transit: So long as you have to keep turning vernier screw down on left to keep body on horizon, the body is rising, and first dip of body below horizon without having turned vernier screw is start of body's fall after transit.

As declination and latitude get farther apart, conditions become more favorable for finding latitude and less so for longitude.

In working the sun for reduction to meridian, t should

not exceed in minutes the number of degrees in sun's meridian zenith distance.

Reduction to the meridian is not nearly so reliable as the meridian sight. Any error in D. R. longitude makes t in error and as the formula uses t^2 the error becomes much greater. When observed altitude is over 60° the procedure is also less reliable.

A rough observation of Polaris may be made as stars begin to fade at morning twilight and this can be made accurate when horizon has become clearer.

Polaris has been observed for latitude during an electrical storm at night.

Azimuth

When Mizar of the Big Dipper or Ruchbah of Cassiopeia are either above or below Polaris, the latter bears true N. and can be used for compass error.

"When it is desired to swing ship using sun azimuths, it is convenient to use a graph laid out on coordinated (cross-section) paper. The interval 5 P. M. to 5:40 P. M., for instance, may be laid off on a horizontal line, a square for each minute, numbering every 5-minute square. The vertical scale, centered on the left-hand end of the horizontal scale, is graduated to the degrees of azimuth, with two squares for each degree. The azimuths taken from the tables for 5 P. M. and 5:40 P. M. (using, as an example, lat. 49° N., and declination 17° 30′ N.) are then plotted and a line drawn connecting them. The azimuth for any intermediate time is now readily available. Suppose the azimuth for 5:11 P. M. is desired: Run up the vertical line of 5:11 until

it intersects the azimuth line, then run horizontally to the left to the intersection with the vertical scale, where the azimuth is indicated as N. 87° 15′ W." (Chapman, p. 225.)

See Dutton, p. 347, for method of obtaining a curve of magnetic azimuths by H. O. 214.

Line of Position

It will sometimes be possible to get a line of position which is parallel to the course from a body abeam. When this is plotted it will at once show which way the ship is being set from the course and how far.

A line of position can sometimes be obtained which is perpendicular to the course from a body dead ahead or astern. This gives a good check on the run.

If a body is observed on the prime vertical (due E. or W.) the line obtained runs N. and S. and is unaffected by any error in the D. R. latitude.

The meridian altitude gives a line running E. and W. which is independent of any longitude error.

If only a single Summer line is obtainable, it may be of great value combined with a terrestrial bearing to give a fix; or the position on a line near shore may be told from soundings; or if the line is parallel to shore it will tell distances from land; or, finally, if it is perpendicular to shore the captain will know what to look for.

Plotting

Draw lines of position for a fix before labeling them so letters will not obscure intersection.

Above a line of position put the time it was obtained,

using four figures for hours and minutes only, as: 0009, 0017, 0231, 1604. This is usually Zone Time.

Below the line, print name of body observed.

When a line has been advanced in a running fix, put above it both the time it was originally obtained and the time of the new line it is to cross. Below, print the name of body observed.

Lines over 5 hours old are not dependable.

Lines under 30 minutes old can be used without calling the resulting fix a running one. (*See* Chap. 15.)

Carry forward position from a fix by simple dead reckoning. Disagreement between this and the next fix is attributed to "current" and the fix starts a new D. R. The D. R. track or course line is drawn according to true course and labeled above with *C* followed by course in degrees and below with *S* followed by speed in knots.

Given a D. R. position and a line of position not running through it and known current:

A perpendicular from D. R. to L. of P. determines Computed Point on the L. of P.

After plotting the current line from D. R., a perpendicular to L. of P. determines Estimated Position on L. of P.

The principles of chart construction, Mercator or polar, should be studied in Chapters I and VI of Dutton.

The Weems Universal Plotting Charts in notebook form are convenient. A compass rose, three unnumbered latitude lines making equal spaces, and a scale for longitude are on each left-hand page. The latitude lines are numbered by the navigator according to the area under consideration and two or more longitude lines are drawn in and numbered, the spaces varying with latitude. Opposite each chart

is a blank page for observational data. Thus all the record is kept easily available and the marking up of regular charts is avoided.

If plotting sheets are not available, the following suggestion is made by Chapman in *Piloting, Seamanship and Small Boat Handling* (1940, p. 220), "A simple method of laying off intercepts in a work book is to employ the rulings of the page as meridians of 1′ of longitude each, crossing them with the parallel of D. R. latitude. From any intersection of a meridian with this parallel draw a line at an angle equal to the degree of latitude. This line becomes a distance scale on which each space between rulings is 1 mile or 1′ of latitude available for use in laying off the intercepts and in measuring the latitude difference between the fix and D. R. latitude. In effect we have a miniature Mercator chart."

25. Navigator's Stars and Planets

THERE ARE MANY good books by which the student can become familiar with stars and with the constellations or groups of stars whose names date back many centuries. A somewhat different presentation has been worked out here for the student navigator. The fifty-four stars of the regular list in the N. A. are taken up in the order given there, that is in order of increasing right ascension from $0^h\ 5^m\ 25^s.7$ to $23^h\ 1^m\ 54^s.2$ (1943). An arbitrary starting time is chosen but the list may be used at any later month providing one identification is made. For each subsequent star is found more eastward and described in relation to one or more close predecessors.

It must be remembered that directions are those on the concavity of the celestial sphere. Curved lines must be imagined in following them. For instance, all directions of arcs of great circles radiating out from the North star are south. A small circle around the pole star followed clockwise as we observe it is always leading east, counterclockwise west.

As each star to be described is farther eastward on the celestial sphere, it is more elevated either later in the same evening or at the same time later in the year. The whole sphere appears to be revolving counterclockwise around the North star as a center (due north and as high in degrees

as your latitude in northern hemisphere) rising in the east and setting in the west (or coming back under the North star in north latitudes) and showing more of itself in the east each night as the earth journeys around the sun.

The easiest way to become familiar with these stars is to study and use a small celestial globe. Set for latitude, date and time, it shows better than any chart or table what stars are spread out for the observer.

In the following list, stars marked with * are of too low declination to be visible from mid-north latitudes as Chicago or New York. Figures following star names are apparent magnitudes.

Stars

1. *α Andromedae (Alpheratz)* 2.2

Looking N. E. about 9 P. M. in mid-August in Lat. 42° N. you see four bright stars in a row slightly sloping up to right, about equal distances apart, with a slight concavity in the row on upper side. The 4th or southmost star is the one named above. The angular distance between these stars is about 15° of the celestial sphere and this is the distance hereafter designated as 1 unit. Alpheratz is the N. E. corner of the "Square of Pegasus," a very easily recognized group with sides each about 15° long and at this time with its S. E. corner pointing down toward the horizon.

2. *β Cassiopeiae (Caph)* 2.4

About ½ way from #1 to the pole star is #2. It is the westernmost of 5 stars forming the letter "W" whose upper surface is toward the pole.

3. *β Ceti (Deneb Kaitos)* 2.2
About 3 units S. by E. of #1.

4. *δ Cassiopeiae (Ruchbah)* 2.8
4th. star to the E. in the "W" mentioned in #2.

*5. *α Eridani (Achernar)* 0.6
About 2½ units S. by E. of #3.

6. *α Arietis (Hamal)* 2.2
1½ units S. of 2nd. star from N. in row of 4 mentioned in #1.

*7. *θ Eridani (Acamar)* 3.4
1½ units N. E. of #5.

8. *α Persei (Marfak)* 1.9
The first or northmost of row of 4 mentioned in #1.

9. *α Tauri (Aldebaran)* 1.1
About 2 units S. E. of 2nd. in row of 4 mentioned in #1 is small dim cluster: the Pleiades. Proceed 1 unit more to #9. Red tinged.

10. *β Orionis (Rigel)* 0.3
2 units S. S. E. of #9. The S. W. corner of a rough rectangle—Orion—whose length is N. and S.

11. *α Aurigae (Capella)* 0.2
1⅓ units E. by S. of #8.

12. *γ Orionis (Bellatrix)* 1.7
1 unit S. E. of #9. The N. W. corner of Orion.

13. *ε Orionis (Alnilam)* 1.8
The middle of 3 stars in center of Orion which are slanting S. E. and N. W.

14. α *Orionis (Betelgeux)* 0.5 to 1.1
The N. E. corner of Orion.

*15. α *Argus (Canopus)* −0.9
3 units S. S. E. of #10.

16. α *Canis Majoris (Sirius)* −1.6
1½ units S. E. of #13. Our brightest star.

17. ε *Canis Majoris (Adhara)* 1.6
1 unit S. by E. of #16. Westernmost of 3 close stars.

18. α *Canis Minoris (Procyon)* 0.5
1¾ units E. of #14.

19. β *Geminorum (Pollux)* 1.2
1½ units N. of #18. Stars 14-18-19 make a right-angled triangle whose hypotenuse faces N. W.

*20. ε *Argus* 1.7
1¼ units S. E. of #15. Westernmost of 3 making equilateral triangle.

*21. λ *Argus (Al Suhail al Wazn)* 2.2
2 units E. N. E. of #15.

*22. β *Argus (Miaplacidus)* 1.8
1¾ units S. of #21. Stars 15-21-22 form an equilateral triangle.

23. α *Hydrae (Alphard)* 2.2
2 units E. S. E. of #18.

24. α *Leonis (Regulus)* 1.3
1½ units N. N. E. of #23. Stars 18-23-24 make a right-angled triangle whose hypotenuse faces N. by W.

25. *α Ursae Majoris (Dubhe)* 2.

3½ units N. by E. of #24. The upper extremity of "big dipper" away from handle. It and star ½ unit S. constitute the "pointers" which point to the pole star. Seven stars in whole dipper. Handle curves down.

26. *β Leonis (Denebola)* 2.2

1⅔ units E. by N. of #24.

*27. *α′ Crucis (Acrux)* 1.6

1⅓ unit E. N. E. of #22. Southmost star in "Southern Cross."

*28. *γ Crucis* 1.6

½ unit N. of #27. Top star in "Southern Cross."

*29. *β Crucis* 1.5

⅓ unit S. E. of #28. East star in "Southern Cross."

30. *ε Ursae Majoris (Alioth)* 1.7

3rd. in handle of "big dipper" counting tip of handle as 1st.

31. *ζ Ursae Majoris (Mizar)* 2.4

2nd. in handle of "big dipper"—next to tip.

32. *α Virginis (Spica)* 1.2

2⅓ units S. E. of #26.

*33. *θ Centauri* 2.3

1⅔ units S. S. E. of #32 or 2 units N. E. of #27.

34. *α Bootis (Arcturus)* 0.2

2 units S. by E. of tip of handle of "big dipper." Stars 26-32-34 make an equilateral triangle with sides 2⅓ units long.

*35. α *Centauri (Rigel Kentaurus)* 0.3
1 unit E. by N. of #27.

36. β *Ursae Minoris (Kochab)* 2.2
1⅔ units N. by E. of tip of handle of "big dipper." The upper extremity of "little dipper" away from handle. Seven stars in whole dipper, the tip of handle being pole star 1 unit away. Handle curves up.

37. α *Coronae Borealis (Alphecca)* 2.3
1⅓ units E. N. E. of #34.

38. δ *Scorpii (Dschubba)* 2.5
2½ units E. S. E. of #32. Center one of 3 similar in row ½ unit long going N. and S.

39. α *Scorpii (Antares)* 1.2
3 units E. S. E. of #32. Reddish.

*40. α *Trianguli Australis* 1.9
1 unit S. E. of #35.

41. η *Ophiuchi m (Sabik)* 2.6
1 unit N. E. of #39.

*42. λ *Scorpii (Shaula)* 1.7
1¼ unit S. E. of #39.

43. α *Ophiuchi (Rasalague)* 2.1
2 units S. E. by E. of #37.

44. γ *Draconis (Etamin)* 2.4
2½ units E. of tip of handle of "big dipper."

*45. ε *Sagittarii (Kaus Australis)* 2.
1¾ unit E. S. E. of #39.

46. *α Lyrae (Vega)* 0.1
1 unit S. E. by S. of #44. In conditions named in #1 this is the bright star almost in zenith.

47. *σ Sagittarii (Nunki)* 2.1
2 units E. of #39.

48. *α Aquilae (Altair)* 0.9
2½ units S. E. by S. from #46.

*49. *α Pavonis* 2.1
2 units S. E. of #45.

50. *α Cygni (Deneb)* 1.3
1½ units E. N. E. of #46. Stars 46-48-50 make a right-angled triangle with hypotenuse facing E. by S.

51. *ε Pegasi (Enif)* 2.5
2 units E. of #48.

*52. *α Gruis (Al Na'ir)* 2.2
1⅓ units N. E. by E. of #49.

53. *α Piscis Australis (Fomalhaut)* 1.3
3 units S. of S. W. star of "Square" (*see* #1).

54. *α Pegasi (Markab)* 2.6
S. W. star of "Square."

Planets

In contrast to stars which usually twinkle, planets shine with a steady light. Instead of being a mere point of light, a navigational planet shows a distinct disc when viewed with binoculars. This disc can sometimes be seen with the unaided eye. The planets are always within a belt 8° each

side of the ecliptic (projection of plane of the earth's orbit or apparent path of the sun). Venus, Mars and Jupiter are brighter than any star. Certain individual characteristics of the four navigational planets are as follows:

Venus is the brightest heavenly body after the sun and moon. It is 12 times as bright as Sirius, the brightest star. Its orbit lies nearer the sun than our earth's and so it is never seen more than about 47° of angular distance from the sun. For the same reason it shows phases, like the moon, which can be easily detected with binoculars and which vary its magnitude or brightness from −4 to −3. It appears as either a "morning" or "evening star." It gets around the sun in 225 of our days. When its R. A. differs by over 2 hours from that of the sun, Venus may be seen in daylight if above horizon.

Mars shines with a reddish light and varies in magnitude from −3 to +2. It must be distinguished from Aldebaran (#9) and Antares (#39) which are both reddish stars of magnitude 1 and near the ecliptic. Its orbit lies next outside the earth's and its year is 687 of our days. Much speculation has been given to the question of whether Mars is inhabited by intelligent beings.

Jupiter is a brilliant white planet of −2 magnitude. Its orbit lies next outside those of the asteroids and its year is almost 12 of our years. Its diameter is over 12 times that of the earth and it has 11 moons. Usually 4 of these can be seen with binoculars. Roemer in 1675 made a remarkably close estimate of the speed of light from observations of the eclipses of these moons.

Saturn is yellowish white and of magnitude 0 to +1. Only two stars are brighter than this planet: Sirius (#16)

and Canopus (#15). It is more easily mistaken for a star than are the other three we have described. But when seen in a good telescope, it is a unique heavenly body because of its "rings." These are layers of billions of whirling particles extending out from its equator. When seen on edge they are a thin line but as the position changes they are seen to be broad bands. There are also 10 moons. Saturn's orbit lies next outside Jupiter's and its year is 29½ of our years. Consequently, we do not notice much shift of its position among the stars between one year and the next. It is so much less dense than the earth that it would float on water.

26. Reference Rules for Book Problems

ON THE PAGE FOLLOWING is Table 19, given for occasional reference only. Learning its contents without understanding each step would be the very sort of thing in the study of navigation which this Primer attempts to prevent. When doing actual work at sea the labeling of angles east or west is usually self-evident or easily determined from a rough diagram of the sort described in Chapter 2. The interested student, however, will want to work out problems for practice which he finds in textbooks. Then it is a more difficult matter to imagine oneself transported to the bridge of a ship, perhaps on the other side of the world, and to visualize the given situation accurately. The table provides a means of checking work done on such problems.

TABLE 19
RULES FOR TIME AND ANGLES

ALL CASES

W (Watch = zone time in A.M. & 12 hrs. behind zone time in P.M.)
+ *C-W* (Add 12 hrs. to *C*, if necessary, to get this)
CF (− 12h. if *13* or over)
± *CC* (& possibly ± 12h. as follows): G.C.T. = zone time + zone number in W. Long. or − zone number in E. long. Add 24h. to zone time before subtracting if zone time is $<$ zone number in E. long. "Longitude West, Greenwich Time Best, Longitude East, Greenwich Time Least" (considering date as well).

Estimate G.C.T. roughly. Then:—

1. If G.C.T. is 12h. $>$ C.F. add 12h. with C.C.
2. If G.C.T. is between 0^h and 1^h subtract 12h. with C.C.
3. If G.C.T. is $>$ 24^h in W. Long. it means excess is time and date is one more.
4. If G.C.T. is $>$ Z.T. in E. Long. it means date is one less.

G.C.T. & date

Sun

G.C.T.
± Eq. T (NA)
+ Corr.
GAT
12h
GHA (= GAT − 12 if sun W. of G or 12 − GAT if sun E of G) E or W

GHA arc (always $<$ 180°) E or W
Long DR E or W
LHA
If GHA & Long have same name:—
LHA = difference
If GHA & Long have different names &
Sum = $<$180°, LHA = sum
Sum = $>$ 180°, LHA = 360°—sum
Observation or diagram usually tells E or W.
Also, by rule, if:—
Same names &
GHA $>$ Long then LHA = same
GHA $<$ Long then LHA = different
Different names &
GHA + Long = $<$180°, LHA = like GHA
GHA + Long = $>$180°, LHA = like Long

Star—Planet—Moon

G.C.T.
+ RAM ⊙ + 12 (GST at 0^h GCT) (NA)
+ Corr. (for amt. GCT $>$ 0^h)
GST (subt. 24 if $>$ 24)
RA (NA)
GHA (= difference & if $>$ 12 subt. it from 24) E or W
Observation or diagram usually tells E or W. Also, by rule, if:—
GST − RA = $<$12 then GHA = W; $>$12 subt. from 24 & GHA = E
RA − GST = $<$12 then GHA = E; $>$12 subt. from 24 & GHA = W

GHA arc (always $<$ 180°) E or W
Long DR E or W
LHA (same as for sun) E or W

Any Body by G.H.A. Method

G.C.T.

GHA arc at — GCT (NA) W
+ corr. (days, hours, minutes)
+ corr. (seconds)
GHA E or W
(If $<$ 180° = W
If between 180° & 360°, subtract from 360° & = E
If $>$ 360°, subtract 360° & = W)
Long DR E or W
LHA (same as for sun) E or W

27. Finding G. C. T. and Date

DESIGNED FOR THE BEGINNER, Table 20 is to be used in checking his calculations of time. It expresses the first part of Table 19 in another way and like that table will be found more useful in doing book problems than necessary in actual work. It gives at a glance the G. C. T. and date from a 12-hour face chronometer at any hour of a 12-hour face watch on zone time in any time zone. Until watches and chronometers are generally equipped with 24-hour dials, this matter will always be a potential source of confusion and error. Meanwhile the table may be used as follows:

Suppose on July 4 you are in Long. 135° E. with no chronometer error. (Table 6 on page 41 shows this to be zone minus 9.) Suppose it is morning and navigator's watch on zone time reads 8 o'clock. Wanted G. C. T. and date. Look in vertical column of −9 zone for figure 8 in A. M. section. Follow horizontally till central chronometer column is reached and figure 11 found. This is, of course, chronometer face. Look at overprinting in the triangle where you started and find "$C + 12$ Day before." Apply this to chronometer, $11 + 12 = 23$, and subtract one from date: G. C. T. is $23^h\ 00^m\ 00^s$ on 3 July.

When the time is between 12 and 1, an additional step is necessary. Say zone time is 12:30 P. M. in zone + 7. This means half an hour after 12 A. M. (upper half). Follow vertical column down to the figure 1 (lower half). Then follow horizontal to center, finding $C = 8$. Subtract 30 minutes. C then becomes 7:30. The overprinting in zone group is $C + 12$. $7{:}30 + 12 = 19{:}30 =$ G. C. T. with no change of date.

TABLE 20
FOR FINDING G. C. T. AND DATE

From a 12-hr. face chronometer at any hour of a 12-hr. face watch on zone time in any time zone.

	W. LONG. ZONES												G	E. LONG. ZONES												
	+12	+11	+10	+9	+8	+7	+6	+5	+4	+3	+2	+1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	
	WATCH												C	WATCH												
A	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	A
A	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	A
A	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	A
A	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	A
A	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	A
A	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	A
M	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	M
M	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	M
M	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	M
M	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	M
M	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	M
M	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	M
P	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	P
P	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	P
P	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	P
P	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	P
P	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	P
P	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	P
M	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	M
M	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	M
M	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	M
M	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	M
M	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	M
M	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	M

NEXT DAY

DAY BEFORE

← INCREASE DATE BY ONE SAILING WEST DECREASE DATE BY ONE SAILING EAST →

Each figure in central vertical column gives corrected chronometer reading for zone time in same horizontal row.

12 in zone groups is only an instant—not a second over. You must follow vertical column down or up to other half to get succeeding hour of 1.

1 in zone group includes the minutes of preceding and succeeding hour.

2 to 11 inclusive in zone groups include the minutes of the succeeding hours.

Overprinting in the eight triangles tells what to do with the corrected chronometer time in order to get G. C. T. and Date.

28. Abbreviations

a.	Altitude difference or intercept.
C.	Course.
C. C.	Chronometer correction.
C. F.	Chronometer face.
C. O.	Chronometer at observation.
C. S.	Chronometer at stop time.
C-W	Chronometer minus watch.
co-L	Co-latitude.
Corr.	Correction.
cos.	Cosine.
cot.	Cotangent.
C. P.	Computed point.
csc.	Cosecant.
D. D.	Daily difference.
d. or Dec.	Declination.
Dep.	Departure.
Dev.	Deviation.
Diff. or D.	Difference of.
Dist.	Distance.
D. R.	Dead Reckoning.
E.	East.
E. P.	Estimated position.
Eq. T.	Equation of Time.
G.	Greenwich meridian.
G.A. T.	Greenwich Apparent Time (or G. A. C. T.)
G. C. T.	Greenwich Civil Time.
G. H. A.	Greenwich Hour Angle.

G. S. T.	Greenwich Sidereal Time.
H.	Meridian altitude.
h.	Altitude.
h_c	Calculated altitude.
h_o	Observed altitude corrected.
h_s	Sextant altitude.
H. A.	Hour angle.
hav.	Haversine.
H. D.	Hourly difference.
H. E.	Height of eye.
H. P.	Horizontal parallax.
I. C.	Index correction.
L. or Lat.	Latitude.
L. A. N.	Local apparent noon.
L. A. T.	Local apparent time (or L. A. C. T.).
L. C. T.	Local civil time.
L. H. A.	Local hour angle (Properly, only W.)
L. S. T.	Local sidereal time.
Lo. or Long.	Longitude.
log.	Logarithm.
M.	Observer's meridian.
m.	Meridional difference of latitude.
Mag.	Magnetic.
M. D.	Minutely difference.
Mid. L.	Middle latitude.
N.	North.
N. A.	Nautical Almanac.
nat.	Natural.
Obs.	Observed.
O. S. M.	The "opposite-the-sun" meridian.
p.	Polar distance.

P.	Parallax.
p. s. c.	Per Standard Compass.
R.	Refraction.
R. A.	Right ascension.
R. A. M. ☉	Right ascension of mean sun.
R. W.	Ran stop-watch.
S.	South.
S. D.	Semidiameter.
sec.	Secant.
sin.	Sine.
t.	Meridian angle. (E. or W. $< 180°$)
tan.	Tangent.
Var.	Variation.
W.	West or Watch.
z.	Zenith distance.
Z.	Azimuth.
Z_n	Azimuth on 360° scale.
☉	Sun.
☾	Moon.
●	Planet.
✳	Star.
♀	Venus.
♂	Mars.
♃	Jupiter.
♄	Saturn.
♈	First Point of Aries or Spring Equinox.
$\sim$	Difference from.
$>$	Greater than.
$<$	Less than.

29. Forms

THE BEGINNER will do better work if he follows a certain form for each procedure. A form for line of position by H. O. 211 has been prepared which I hope leaves nothing to chance. Roughly it is divided into three parts. The first covers the sextant and timepiece work and general data; the middle part is done by means of the *Nautical Almanac;* and the last section is worked entirely by the 211 Table. Course and log reading should be noted in a moving ship to allow for run between, in case another sight is taken later for a fix and the first line has to be moved up. The stop-watch method is suggested as described at the end of Chapter 2. A space is provided for everything which may be needed, no matter what heavenly body is used. The small squares for D. D., H. D., M. D. and H. P. are for values which are found or calculated during the first consultation of the N. A. for the body being used and mean daily difference, hourly difference, minutely difference and horizontal parallax. The proper entries should be made here before turning to the various correction tables in other parts of the almanac so that no looking back will be necessary. The abbreviations in the altitude correction section are: *R,* refraction; *P,* parallax; *S D,* semidiameter; *H E,* height of eye; *I C,* index correction, etc. After the student has used a form of this sort for some time, he will have become so familiar with the various things to remember that he may then advantageously arrange matters in a more abbreviated way.

There are three other forms, designed for H. O. 211, which may be found occasionally convenient. A form for line of position by the cosine-haversine method with time and altitude azimuth should be useful when Bowditch alone is available. The remarkable, though at present rather impractical, formulas for Aquino's Fix may be easily applied in a form here offered.

A form for Dead Reckoning, two for Middle Latitude and two for Mercator sailing, should help the beginner.

Forms for Day's Run and Day's Current show how each may be worked either by logs or by Table 3.

When a word is followed by a different word in parentheses, as Dep. (Lat.), it means: find the first in the column headed by the word in parentheses.

LINE OF POSITION

H. O. 211

(Form by J. F.)

SHIP

EN ROUTE TO COURSE

D.R. POSITION : LAT. LONG. LOG

DATE TIME OF DAY

BODY OBSERVED

BEARING APPROXIMATELY

SEXTANT ALTITUDE

INDEX CORRECTION

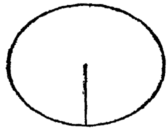
HEIGHT OF EYE

C.S.

R.W.= ____________

C.O.

C.C. ± ____________



M

DATE _

GREENWICH HOUR ANGLE		DECLINATION	ALTITUDE (−)	ALTITUDE (+)
AT _ _h		AT ______h		
+FOR _ _h		+FOR ______h	HE	Hs
+FOR _ _m		+FOR ______m	★●R	⊙RPSD
+FOR _ _s_		+FOR ______m ______	⊙SD	⊙SD
G.H.A. =	W	DEC. =	☾RPSD	☾RPSD
IF > 180° =				IC
LONG.D.R.=			TOTAL	TOTAL
H.D.☾	SUBTRACT LIKES / ADD OPPOSITES / IF > 180° SUBTRACT FROM 360°	H.D.⊙☾	H.P.☾	LESS
M.D.●		D.D.●		Ho

L.H.A.

DEC. B

R ---- Ⓐ+B

K (NAME OF DEC) -------+Ⓐ

LAT. D.R.

K~L (OPPOSITES + / LIKES −) ---→ |B

Hc ------------Ⓐ Ⓐ+B

Ho

a {TOWARDS / AWAY} MILES

Ⓐ→ FIND NO. IN A & TAKE OUT WHAT → POINTS TO

IF L.H.A. > 90° TAKE K FROM BOTTOM

IF K SAME NAME AS & > LAT. TAKE Z FROM TOP Z

IF K NEAR 90° MUST INTERPOLATE Zn

Latitude

H. O. 211

(Favill)

t	A		
	$+$		
d	B	A	
h			A
		$-$	$-$
$R \longrightarrow$	A	B	B
ϕ''	$\longleftarrow$	A	
ϕ'	$\longleftarrow$	$\longleftarrow$	B
Lat.			

Give ϕ'' same name as declination
ϕ' is N if body bears N & E or N & W
ϕ' is S if body bears S & E or S & W
Combine by adding if different, subtracting if alike
$L = \phi' \pm (180° - \phi'')$ for lower transit

Longitude
H. O. 211
(Hinkel)

h		$90° = 89°\ 60'.0$
$+$		
L	B	$\pm d$
$+$	$+$	
p	A	p
$2)$	l.v.	
s	B	
$-h$	$+$	
$s-h$	A	
	l.u.	
	$-$	
	l.v.	

l.csc $^2\ \frac{1}{2}\ t =$	& $\div\ 2 =$
l.csc $\frac{1}{2}\ t =$	& found in A gives
$\frac{1}{2}\ t =$	& $\times\ 2 =$
$t =$	& combined with
G.H.A.,	gives
Longitude	

When d & L have same name:

$$p = 90° - d$$

When d & L have opposite names:

$$p = 90° + d$$

Altitude Azimuth

H. O. 211

(Hinkel)

h	B	$90° = 89°\ 60'\ .0$
$+$	$+$	
L	B	$\pm d$
$+$		
p	l.v.	p
2)		
s	B	
$-p$	$+$	
$s-p$	B	
	l.u.	
	—	
	l.v.	
l.sec^2 ½ $Z =$		& ÷ 2 =
l.sec ½ $Z =$		& found in B gives
½ $Z =$		& × 2 =
Z		

When d & L have same name:

$$p = 90° - d$$

When d & L have opposite names:

$$p = 90° + d$$

Line of Position
H. O. 9
(Bowditch: Cosine-Haversine Altitude, and Time and Altitude Azimuth)

t	l.hav.	l.sin.
	+	
L	l.cos.	+
	+	
d	l.cos.	l.cos.
	θ l.hav.	
	θ n.hav.	
	+	
L ~ *d*	n.hav.	+
z	←z n.hav.	
From 89° 60′ .0		
hc		l.sec.
ho		*Z* l.sin.
		Z = from? pole

Aquino's Fix
(Form modified by J.F.)
From Simultaneous Altitude and Gyro Azimuth

ho	l.sec.	l.tan.		
	+	+		
Zg	l.csc.	l.sec.		89° 60′ .0
				−
a — — — — — →	l.csc.	B.l.tan.	B =	
	−			
d	l.sec.	l.tan.	C =	
		+		
t	←l.csc.	l.sec.	±	
G.H.A.		b.l.tan.	b =	
Long.			Lat.	

Middle Latitude Sailing

FOR COURSE AND DISTANCE

	Latitude		*Latitude*		*Longitude*
From	° ′ ″		° ′ ″		° ′ ″
To	° ′ ″		° ′ ″		° ′ ″
DL	° ′ ″	2)	° ′ ″	*DLo*	° ′ ″
			° ′ ″		′
=	′	*Lm*		=	
DLo	′	log			
Lm	° ′ ″	l.cos.			
(Dep.)		log			
DL	′	−log		log	
Course		l.tan		l.sec	
Distance				log	

FOR POSITION REACHED

Distance		log	log	
Course		l.cos.	l.sin.	
DL	′	log	log	(Dep.)
=	° ′ ″			
L_1	° ′ ″			
L_2	° ′ ″			
$L_1 + L_2$	° ′ ″ ÷ 2 =			
Lm	° ′ ″		l.sec.	
DLo	′		log	
=	° ′ ″			
Lo_1	° ′ ″			
Lo_2	° ′ ″			

DEAD RECKONING

TIME	P. LOG.	VAR.	DEV.	WIND	LEE-WAY	TOTAL ERROR	COURSE		DIST.	D. LAT.		DEP.	
							P.S.C.	TRUE		N	S	E	W

D.L. N/S	TABLE 3 MID. L. (COURSE)		TOTALS				
L_1 N/S	DEP. (LAT.)		SUBT. SMALLER				
L_2 N/S	D.LO. (DIST.) ′		D.L. & DEP.				
	D.LO. ° ′ ″	E/W	↓	COURSE			
L_1+L_2 ÷2	LO_1	E/W	TABLE 3 →				
MID. L	LO_2	E/W		DISTANCE			

Mercator Sailing

FOR COURSE AND DISTANCE

Latitude					*Longitude*			
From	°	′	″	M.P.		°	′	″
To	°	′	″	M.P.		°	′	″
DL	°	′	″	*m*	*DLo*	°	′	″
=		′			=		′	
DLo		′		log				
m				−log				
Course				l.tan.	l.sec.			
DL					log			
Distance					log			

FOR POSITION REACHED

Distance				log	
Course				l.cos.	l.tan.
DL		′		log	
=	°	′	″		
L_1	°	′	″	M.P.	
L_2	°	′	″	M.P.	
				m	log
DLo		′			log
=	°	′	″		
Lo_1	°	′	″		
Lo_2	°	′	″		

Day's Run

	Latitude		*Longitude*	
By obs. or D.R. noon yesterday		N/S		E/W
By obs. or D.R. noon today		N/S		E/W
	DL	N/S	*DLo*	E/W
	=	′	=	′

By Logs and Mid. Lat. Sailing

DLo ______	log	
Lm ______	l.cos.	
(Dep.)	log	
DL ______	—log	log
Course ______	l.tan.	l.sec.
In ____ Quad = ____		
Dist. ______		log

By Traverse Table and Mid. Lat. Sailing

DLo (Dist.) ______
Lm (Course) ______ } Table 3: Dep. (Lat.) ______

DL (Lat.) ______
Dep. ______ } Table 3: { Course ______ / Dist. ______

Day's Current

(Given D.R. run in miles: *DL* ^N_S and Dep. ^E_W)

	Latitude		*Longitude*	
Position by obs. noon yesterday		^N_S		^E_W
Run by D.R. noon today	*DL*	^N_S	*DLo*	^E_W (from below)
Position by D.R. noon today		^N_S		^E_W
Position by obs. noon today		^N_S		^E_W
Current	*DL*	^N_S (miles)	*DLo*	^E_W (minutes of *Lo*)

By Logs and Mid. Lat. Sailing

DLo, Cur._____	log	Dep. D.R._____	log
*Lm*_____	l.cos.	*Lm*_____	l.sec.
(Dep.)	log	*DLo,* D.R. enter above	log
DL, Cur._____	—log		log
Course, Cur._____	l.tan.		l.sec.
In___Quad._____	= Set		
Dist. Cur._____			log
÷ 24_____	= Drift (m.p.h.)		

By Traverse Table and Mid. Lat. Sailing

Lm (Course)_____ / Dep., D.R. (Lat.)__ } Table 3: *DLo,* D.R. (Dist.)__________ enter above

Lm (Course)_____ / *DLo,* Cur. (Dist.)__ } Table 3: Dep., Cur. (Lat.)__________

DL, Cur. (Lat.)____ / Dep., Cur. (Dep.)__ } Table 3: { Course__________ = Set / Dist.__________

÷ 24__________ = Drift (m.p.h.)

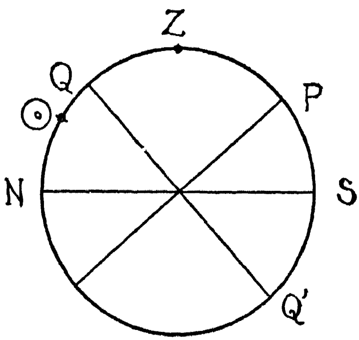
30. Problems

IN CONTRAST to most texts, this Primer has not included problems with its explanations of the various procedures of celestial navigation. The reason for this is that it was hoped the student would more quickly acquire sound principles and a clear understanding of the subject as a whole if he were spared the detail of computations while first reading the book through as a sort of survey course. Plenty of problems can be found in Bowditch or Dutton or other texts and short-cut systems. Best of all are the problems one goes out and sets for himself. However, a few illustrative examples will now be given of the more important procedures. I have taken data for the meridian latitude sights from Bowditch, 1933 edition, and for the time conversions from the 1939 *Nautical Almanac*. The rest are my own. The fact that most of my sights were taken from known positions may lessen interest but has the advantage of showing the degree of accuracy of the results.

Latitude by Meridian Altitude

Case 1: L & d opposite names. $L = z - d$

At sea, May 15, 1925, in Long. 0°, the observed meridian altitude of the sun's lower limb was 30° 13′ 10″; sun bearing north; I. C., + 1′ 30″; height of the eye, 15 feet.

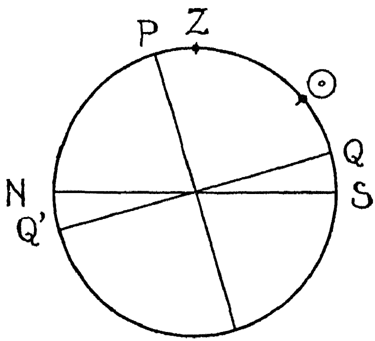


hs	30°	13′	10″		
Corr.	+	12′	02″		
ho	30°	25′	12″		Now subtracting from 90° or
	89°	59′	60″		gives
z	59°	34′	48″		Now from G.C.T. and N.A. we get
d	18°	48′	30″	N.	and $z - d$ gives
L	40°	46′	18″	S.	

Latitude by Meridian Altitude

Case 2: L & d same name and $L > d$. $L = z + d$

At sea, June 21, 1925, in Long. 60° W., the observed meridian altitude of the sun's lower limb was 40° 04′; sun bearing south; I. C., + 3′ 0″; height of the eye, 20 feet.

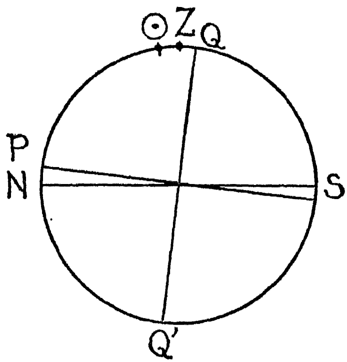


hs	40° 04′ 00″		
Corr.	+ 13′ 21″		
ho	40° 17′ 21″		Now subtracting from 90° or
	89° 59′ 60″		gives
z	49° 42′ 39″		Now from G.C.T. and N.A. we get
d	23° 26′ 48″	N.	and $z + d$ gives
L	73° 09′ 27″	N.	

Latitude by Meridian Altitude

Case 3: L & d same name and $d > L$. $L = d - z$

At sea, April 14, 1925, in Long. 140° E., the observed meridian altitude of the sun's lower limb was 81° 15′ 30″; sun bearing north; I. C., − 2′ 30″; height of the eye, 20 feet.



hs	81° 15′ 30″		
Corr.	+ 9′ 00″		
ho	81° 24′ 30″		Now subtracting from 90° or
	89° 59′ 60″		gives
z	8° 35′ 30″		Now from G.C.T. and N.A. we get
d	9° 10′ 48″	N.	and $d - z$ gives
L	0° 35′ 18″	N.	

Latitude by Meridian Altitude

Case 4: L & d same name, lower transit.

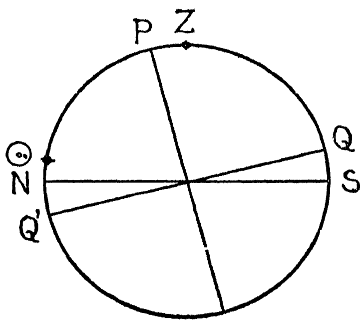
$$L = 180° - (d + z) = h + p$$

June 13, 1925, in Long. 65° W., and in a high northern latitude, the observed meridian altitude of the sun's lower limb was 8° 16′ 10″, sun below the pole; I. C., 0′ 00″; height of the eye, 20 feet.

hs	8° 16′ 10″		
Corr.	+ 5′ 11″		
ho	8° 21′ 21″		Now subtracting from 90° or
	89° 59′ 60″		gives
z	81° 38′ 39″		Now from G.C.T. and N.A. we get
d	23° 10′ 56″	N.	and $d + z =$
	104° 49′ 35″		and subtracting from 180° or
	179° 59′ 60″		gives
L	75° 10′ 25″	N.	

or

90°	89° 59′ 60″	
d	−23° 10′ 56″	
p	66° 49′ 04″	
ho	+ 8° 21′ 21″	
L	75° 10′ 25″	N.



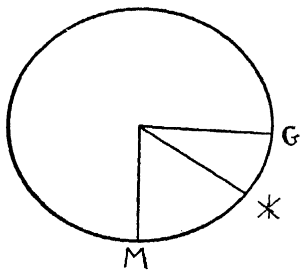
Latitude by Phi Prime, Phi Second
(H. O. 211)

At anchor, in evening of August 21, 1935, in Lat. 42° 12′ N., Long. 87° 48′ W., observed star Deneb, bearing about ENE, as follows: sextant altitude 58° 56′; I. C. 0′ 0″; H. E. 12 feet; G. C. T. 22 August, $1^h\ 39^m\ 20^s$.

G.H.A. at 1 Aug. 0^h	358° 51′.4
+ for 22^d	20° 41′.9 *
+ for $1^h\ 39^m$	24° 49′.1
+ for 20^s	5′.0
	404° 27′.4 W.
	−360° 00′.0
G.H.A.	44° 27′.4 W.
Long.	87° 48′.0 W.
L.H.A.	43° 20′.6 E.

d = 45° 3′.1 N

R	−0′.6
H.E.	−3′.4
	−4′.0
hs	58° 56′.
ho	58° 52′



t 43° 20′.5 E	A 16346			
	+			
d 45° 3′.1 N	B 15089		A 15014	
ho 58° 52′				A 6754
				−
	31435 A		→ B 5822	B 5822
ϕ'' 54° 1′.5 N			← A 9192	
ϕ' 11° 49′.5 N				← B 932
L 42° 12′ N				

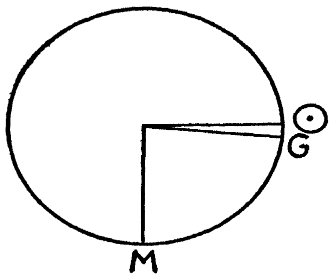
* Beginning with the 1936 N. A., the G. H. A. of stars is given for each day of the month, thus shortening this calculation by 1 step.

Longitude by Time-Sight
(H. O. 211)

On Lake Michigan shore, in morning of June 24, 1935, in about Lat. 42° 12′ N., Long. 87° 48′ W., observed sun's lower limb bearing about E., as follows: sextant altitude 14° 10′; I. C. 0′ 0″; H. E. 87 feet; G. C. T. 24 June, 11^h 46^m 0^s.

G.H.A. at 10^h	329° 29′.6	d = 23° 26′ N	R.P.S. +	12′.1
+ for 1^h 46^m	26° 30′.0	from 90° 00′	H.E. −	9′.1
G.H.A.	355° 59′.6 W.	p = 66° 34′		+ 3′.0
from	360° 00′.0		hs	14° 10′.
G.H.A.	4° 0′.4 E.		ho	14° 13′

h	14° 13′		
+			
L	42° 12′	B	13030
+		+	
p	66° 34′	A	3738
	2)122° 59′	l.v.	16768
s	61° 29′.5	B	32122
	−14° 13′	+	
s-h	47° 16′.5	A	13394
		l.u.	45516
		−	
		l.v.	16768



l.csc² ½ t =	28748	& ÷ 2 =
l. csc ½ t =	14374	& found in A gives
½ t =	45° 54′.5	& × 2 =
t =	91° 49′.0 E.	& combined with
G.H.A.	4° 0′.4 E.	gives
Long.	87° 48′.6 W.	

Altitude Azimuth
(H. O. 211)

At anchor, in morning of June 16, 1936, in Lat. 42° 12′ N., Long. 87° 48′ W., observed sun's lower limb, bearing about E., as follows: sextant altitude 33° 31′ 10″; I. C. 0′ 0″; H. E. 12 feet; G. C. T. 16 June, $13^h\ 32^m\ 6^s$.

---

d =	23° 21′.6 N	R.P.S.	+ 14′.5
from	90° 00′.0	H.E.	− 3′.4
p =	66° 38′.4		+ 11′.1
		hs	33° 31′.2
		ho	33° 42′.3

---

h	33° 42′.3	B	7993
+		+	
L	42° 12′	B	13030
+			
	66° 38′.4	l.v.	21023
	2) 142° 32′.7		
s	71° 16′.3	B	49346
	−66° 38′.4	+	
s-p	4° 37′.9	B	142
		l.u.	49488
		−	
		l.v.	21023
l. sec² ½ Z		28465	& ÷ 2 =
l. sec ½ Z		14232	& found in B gives
½ Z =		43° 54′	& × 2 =
Z =		N. 87° 48′ E.	

(Merely a coincidence that this = Longitude)

Azimuth by H. O. 214

Make 5 columns headed by: Decimals, *t*, *d*, *L*, and Base.

Data taken from Sun problem, which follows:

t 19° 47′.7 E., *d* 23° 26′.8 N., *L* 42° 12′ N.

Enter these as degrees and decimals in left column.

Use H. O. 214, Vol. V, entering with *t* 19°, *d* 23°, *L* 42° (the next lower whole degrees) on page for *L* and *d* of same name and get Base *Z* 134° .3.

Enter this at top of the four remaining columns.

Use H. O. 214 with base *d* & *L* but 1° greater *t*
" " " " " *t* & *L* " 1° " *d*
" " " " " *t* & *d* " 1° " *L*

and write resulting azimuths under Base in proper columns.

Find differences between above and Base which give changes for 1° increase of each variable.

Mark results (+) or (—) according as azimuth is increasing or decreasing with the higher entry figures.

Convert each to minutes of arc.

Multiply each by the decimal fraction of a degree by which corresponding quantity of given data exceeds quantity used to get Base.

Mark results (+) or (—) as before.

Take algebraic total for correction.

Apply to base *Z* in 5th column for *Z* (on 180° system).

Convert to *Z* on 360° system = *Zn*.

TABLE 21

WORKING FORM FOR AZIMUTH BY H.O. 214

Decimals	t	d	L	Base
t 19°.8 d 23°.45	(19°) 134°.3 (20°) 132°.5	(23°) 134°.3 (24°) 132°.8	(42°) 134°.3 (43°) 135°.8	134°.3 −1°.8(Corr.)
L 42°.2	(−) 1°.8 = 108′ × .8 (−) 86′.4	(−) 1°.5 = 90′ × .45 (−) 40′.50 (−) 86′.4 (Add.) (−) 126′.9 (+) 18′. (Sub.) (−) 108′.9 = 1°48′.9 = (−) 1°.8 = Corr.	(+) 1°.5 = 90′ × .2 (+) 18′.0	N 132°.5 E = Z 132°.5 = Zn

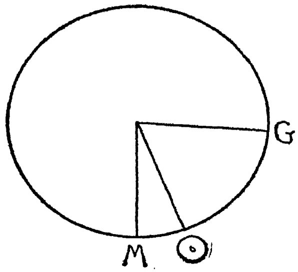
Line of Position from Sun
(H. O. 211)

At anchor, in forenoon of June 21, 1936, in about Lat. 42° 12′ N., Long. 87° 47′ W., observed sun's lower limb, bearing about SSE, as follows: sextant altitude 64° 52′ 10″; I. C. 0′ 0″; H. E. 12 feet; G. C. T. 21 June, 16^{h} 33^{m} 34^{s}.

G.H.A. at 16^{h}	59° 35′.8	d = 23° 26′.8 N.	R.P.S.	− 3′.6
+ for 33^{m}	8° 15′		H.E.	+ 15′.6
+ for 34^{s}	8′.5			
				+ 12′
G.H.A.	67° 59′.3 W.		hs	64° 52′.2
Long. D.R.	87° 47′.0 W.			
			ho	65° 4′.2
L.H.A.	19° 47′.7 E.			

		(+)	(−)	(+)	(−)
t	19° 47′.7	A 47031			
d	23° 26′.8 N	B 3744	A 40017		
		50775 A →	B 2204	B 2204	A 50775
K	24° 45′ N		← A 37813		
L	42° 12′ N				
K∽L	17° 27′			B 2046	
hc	65° 4′.0			← A 4250	→ B 37514
ho	65° 4′.2				A 13261
a	.2 mile toward				↓ Zn 132° 32′

(See diagram on opposite page)



Line of Position from Sun
(H. O. 9)

Previous problem worked by Cosine-Haversine method, with "time and altitude" azimuth formula.

t	19° 47′.7	l. hav.	8.47052	l. sin.	9.52975
		+			
L	42° 12′ N	l. cos.	9.86970	+	
		+			
d	23° 26′.8 N	l. cos.	9.96258	l. cos.	9.96258
		θ l. hav.	8.30280		
		θ n. hav.	.02008		
		+			
L∽d	18° 45′.2	n. hav.	.02654		
z	24° 56′ 15″ ←	z n. hav.	.04662	+	
from	89° 59′ 60″				
hc	65° 3′.7			l. sec.	10.37506
ho	65° 4′.2			Z l. sin.	9.86739
a	.5 mile toward			Z = 47° 28′ (from S. toward E.)	

(This method requires 9 consultations of the tables, whereas H.O. 211 requires but 7.)

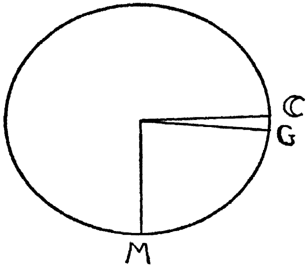
Line of Position from Moon
(H. O. 211)

At anchor, in evening of October 29, 1936, in Lat. 42° 12′ N., Long. 87° 48′ W., observed moon's lower limb, bearing about ENE, as follows: sextant altitude 7° 58′ 40″; I. C. 0′ 0″; H. E. 12 feet; G. C. T. 29 October, 23^h 4^m 1^s. *(See diagram on opposite page.)*

	H.D. 14° 24′.4		H.D. 11′.4		H.P. 61′.5
G.H.A. at 23^h	354° 32′.2	d at 23^h	16° 5′.2 N	R.P.S.	+ 70′.7
+ for 4^m	57′.6	+ for 4^m 1^s	1′	H.E.	− 3′.4
+ for 1^s	.2				
			16° 6′.2 N		+ 67′.3
G.H.A.	355° 31′ W.			hs	7° 58′.6
from	360° 00′				
				ho	9° 5′.9
G.H.A.	4° 29′ E.				
Long.	87° 48′ W.				
L.H.A.	92° 17′ E.				

		(+)	(−)	(+)	(−)
t	92° 17′	A 34.5			
d	16° 6′.2 N	B 1738	A 55694		
		1772.5A →	B 55289	B 55289	A 1772
K	97° 49′ N		← A 405		
L	42° 12′ N				
K∼L	55° 37′			B 24816	
hc	9° 5′.8			← A 80105	→ B 550
ho	9° 5′.9				A 1222
a	.1 mile toward				Zn 76° 28

(This calculation illustrates the exception to Rule 1: since t > 90°, take *K* from bottom; and the exception to Rule 2: since *K* is same name as and > *L*, take *Z* from top. Whenever the first exception applies, the second will also be called for. *See* Fig. 40, page 136.)



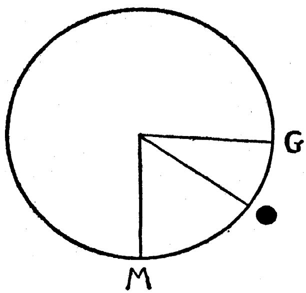
(This diagram is to be used with the Problem on Page 236.)

Line of Position from Planet
(H. O. 211)

On board S.S. *Western States,* making passage Chicago to Mackinac, in evening of July 15, 1937, in D. R. position Lat. 43° 30′ N., Long. 86° 51′ W., observed planet Jupiter, bearing about SE, as follows: sextant altitude 7° 13′; I. C. 0′ 0″; H. E. 35 feet; G. C. T. 16 July, 2^h 7^m 9^s.

	M.D. 15′.0468		D.D. 1′.4		
G.H.A. at 0^h	359° 14′.1	d at 0^h	21° 56′.9 S.	R	− 7′.2
+ for 2^h	30° 4′.8	+ for $2^h 7^m 9^s$	.1	H.E.	− 5′.8
+ for 7^m	1° 45′.3				
+ for 9^s	2′.3	d =	21° 57′ S.		− 13′
				hs	7° 13′
	391° 6′.5				
	− 360° 0′.0			ho	7°
G.H.A.	31° 6′.5 W.				
Long.	86° 51′ W.				
L.H.A.	55° 44′.5 E.				

		(+)	(−)	(+)	(−)
t	55° 44′.5	A 8275			
d	21° 57′	B 3268	A 42736		
		11543 A →	B 19238	B 19238	A 11543
K	35° 36′ S		← A 23498		
L	43° 30′ N				
K∽L	79° 6′			B 72332	
hc	6° 58′.5			← A 91570	→ B 323
ho	7° 0′.0				A 11220
a	1.5 miles toward				↓ Zn 129° 26′



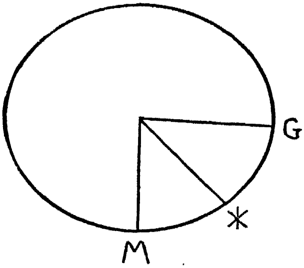
Line of Position from Star
(H. O. 211)

At anchor, about an hour before midnight of October 29, 1936, with full moon, in Lat. 42° 12′ N., Long. 87° 48′ W., observed star Capella, bearing about ENE, as follows: sextant altitude 51° 51′ 40″; I. C. 0′ 0″; H. E. 12 feet; G. C. T. 30 October, $4^h\ 53^m\ 31^s$.

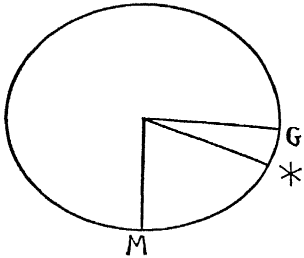
G.H.A. at 0^h	320° 14′.8	d = 45° 56′.1 N.	R	− 0′.8
+ for $4^h\ 53^m$	73° 27′		H.E.	− 3′.4
+ for 31^s	7′.8			
				− 4′.2
	393° 49′.6		hs	51° 51′.7
	− 360° 00′.0			
			ho	51° 47′.5
G.H.A.	33° 49′.6 W.			
Long.	87° 48′.0 W.			
L.H.A.	53° 58′.4 E.			

		(+)	(−)	(+)	(−)
t	53° 58′.4	A 9218			
d	45° 56′.1	B 15771	A 14355		
		24989 A →	B 8259	B 8259	A 24989
K	60° 21′ N	←	A 6096		
L	42° 12′ N				
K ∽ L	18° 9′			B 2216	
hc	51° 47′.0			← A 10475	→ B 20856
ho	51° 47′.5				
					A 4133
a		.5 miles toward			↓
					Zn 65° 24′

(This calculation illustrates the exception to Rule 2: since *K* is same name as and > *L*, take *Z* from top. *See* Fig. 40, page 136.)



(For Problem on Page 239)



(For Aquino's Fix)

Aquino's Fix

At anchor, in evening of June 20, 1936, in Lat. 42° 12′ N., Long. 87° 48′ W., observed star Vega as follows: sextant altitude 41° 16′ 30″; simultaneous gyro azimuth 70° 50′; I. C. 0′ 0″; H. E. 12 feet; G. C. T. 21 June, $2^h\ 7^m\ 9^s$.

(As I had no gyro, this azimuth was actually determined by calculation, and the above is merely to illustrate the method which follows.)

G.H.A. at 0^h	350° 18′.2	d = 38° 43′.4 N.	R	− 1′.2
+ for $2^h\ 7^m$	31° 50′.2		H.E.	− 3′.4
+ for 9^s	2′.3			− 4′.6
	382° 10′.7		hs	41° 16′.5
	− 360° 00′.0		ho	41° 11′.9
G.H.A.	22° 10′.7 W.			

ho	41° 12′	l. sec 12354	l. tan 9.94222	
		+	+	
Zg	70° 50′	l. csc 2477	l. sec 48371	89° 60′.0
				−
a		l. csc 14831	B l. tan 0.42593	B = 69° 26′.5
		−		
d	38° 43′	l. sec 10777	l. tan 9.90397	C = 20° 33′.5
			+	
t	65° 37′.7 E. ←	l. csc 4054	l. sec 38436	+
G.H.A.	22° 10′.7 W.		b. l. tan 0.28833	b = 62° 46′
Long.	87° 48′.4 W.			Lat. 42° 12′.5 N.

Fix from Two Stars *
(H. O. 214)

In evening of Nov. 2, 1941, in D. R. position Lat. 42° 15′ N., Long. 87° 42′ W., observed stars as follows:

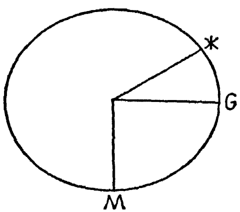
Capella: bearing about N. E. by N., sextant altitude 11° 30′ 00″, G. C. T. Nov. 3, $0^h\ 1^m\ 30^s$.

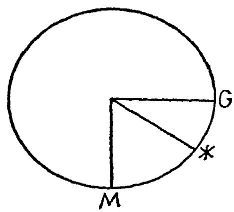
Deneb Kaitos: bearing about S. E. by E., sextant altitude 11° 04′ 50″, G. C. T. Nov. 3, $0^h\ 3^m\ 22^s$.

In each case: I. C. 0′ 0″, H. E. 12 feet.

ALTITUDE CORRECTIONS

Capella		*Deneb Kaitos*
11° 30′ .0	hs	11° 04′ .8
(−) 4′ .7	R	(−) 4′ .9
(−) 3′ .4	H.E.	(−) 3′ .4
11° 21′ .9	ho	10° 56′ .5





* A practice sight from known position, 42° 12′ N., 87° 48′ W., on Lake Michigan shore, using an imaginary D.R. position to illustrate the method.

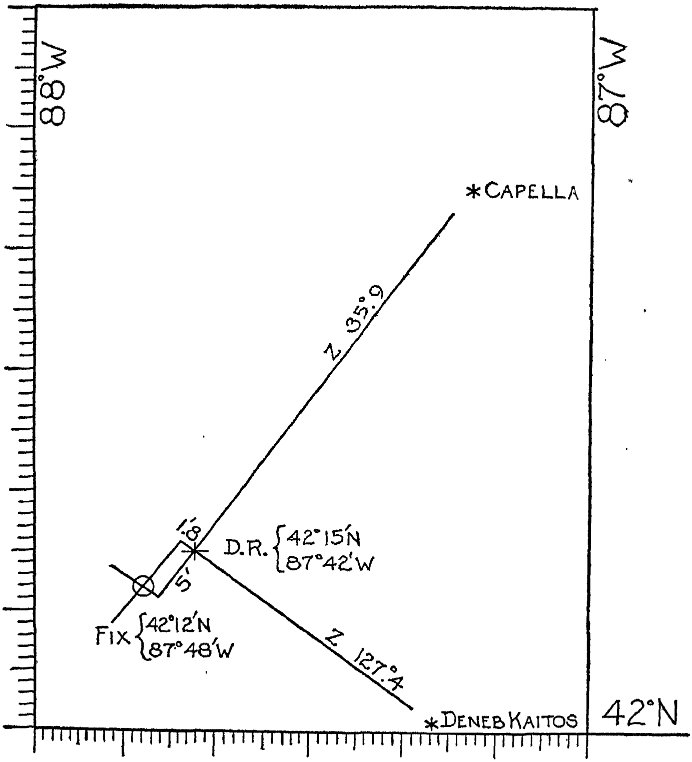


FIG. 45. Fix bv H. O. 214. D. R. Position. (*See opposite page.*)

Solution using the D.R. position and 3 altitude corrections for each body.

(from N.A.)

Capella

d = 45° 56′.2 N

G.H.A. at 0^h = 323° 45′.4 W
+ for 1^m 15′
+ for 30^s 7′.5

324° 07′.9 W
from 359° 60′.0

G.H.A. at obs. 35° 52′.1 E, add
Lo.D.R. 87° 42′.0 W

123° 34′.1 E

Deneb Kaitos

d = 18° 18′.3 S

G.H.A. at 0^h = 31° 41′.4 W
+ for 3^m 45′.1
+ for 22^s 5′.5

G.H.A. at obs. 32° 32′.0 W, from
Lo.D.R. 87° 42′.0 W

55° 10′.0 E

(from H.O. 214, Vol. V.)

Capella

(p. 73)

Enter with	Take out
L 42° N	h 11° 06′.5
d 46° N	△ d 78, △ t 43
t 124°	Z 35°.9

Alt. corr. for:
L (15′ & Z 36) = 12′.1 (+)
d (3′.8 & 78) = 2′.9 (−)
t (25′.9 & 43) = 11′.2 (+)

20′.4 (+)
h from table 11° 06′.5

hc for D.R. 11° 26′.9
ho 11° 21′.9

(away) 5.0 miles

Deneb Kaitos

(p. 63)

Enter with	Take out
L 42° N	h 11° 27′.4
d 18° S	△ d 78, △ t 59
t 55°	Z 127°.4

Alt. corr. for:
L (15′ & Z 127) = 9′.0 (−)
d (18′.3 & 78) = 14′.2 (−)
t (10′ & 59) = 5′.9 (−)

29′.1 (−)
h from table 11° 27′.4

hc for D.R. 10° 58′.3
ho 10° 56′.5

(away) 1.8 miles

(Plotted from D.R. position: 42° 15′ N & 87° 42′ W gives fix at 42° 12′ N & 87° 48′ W.)

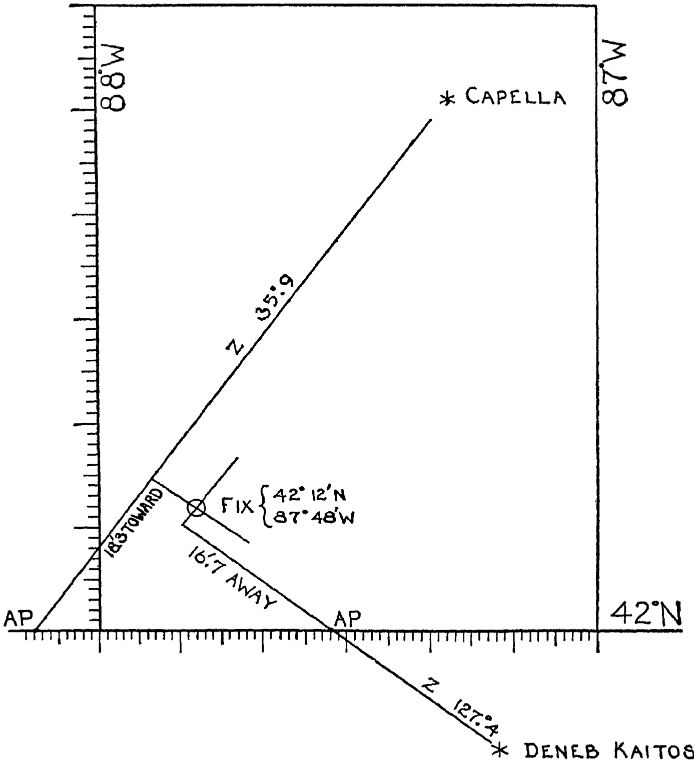


Fig. 46. Fix by H. O. 214. Assumed Position. (*See opposite page.*)

Solution using an assumed position and 1 altitude correction for each body.

Capella			*Deneb Kaitos*	
d =	45° 56′.2 N	(from N.A.)	d =	18° 18′.3 S
G.H.A. at 0^h =	323° 45′.4 W		G.H.A. at 0^h =	31° 41′.4 W
+ for 1^m	15′		+ for 3^m	45′.1
+ for 30^s	7′.5		+ for 22^s	5′.5
	324° 07′.9 W			
from	359° 60′.0			
G.H.A. at obs.	35° 52′.1 E, add		G.H.A. at obs.	32° 32′.0 W, from
Lo assumed	[illegible]° 07′.9 W	(to give *t* a whole no.)	Lo assumed	87° 32′.0 W
	124° E		t	55° E

(from H.O. 214, Vol. V.)

(p. 73) Enter with	Take out		(p. 63) Enter with	Take out
L 42° N	h 11° 06′.5		L 42° N	h 11° 27′.4
d 46° N	△ d 78		d 18° S	△ d 78
t 124°	Z 35°.9		t 55°	Z 127°.4

Alt. corr. for:			Alt. corr. for	
d (3′.8 & 78) =	2′.9 (−)		d (18′.3 & 78) =	14′.2 (−)
h from table	11° 06′.5		h from table	11° 27′.4
hc for A.P.	11° 03′		hc for A.P.	11° 13′.2
ho	11° 21′		ho	10° 56′.5
a = (toward)	18.3 miles		(away)	16.7 miles

Plotted from A.P.	Fix at	Plotted from A.P.
42° N & 88° 07′.9 W	42° 12′ N & 87° 48′ W	42° N & 87° 32′.0 W

Identification
(H. O. 127)

At anchor, in evening of May 5, 1937, in Lat. 42° 12′ N., Long. 87° 48′ W., observed an unknown star as follows: sextant altitude 39° 9′ 10″; I. C. 0′ 0″; H. E. 12 feet; azimuth about 99°; G. C. T. 6 May, $1^h\ 32^m\ 4^s$.

R	−	1′ .2
H. E.	−	3′ .4
	−	4′ .6
hs	39°	9′ .2
ho	39°	4′ .6

1. Entering H. O. 127 with L 42°, Z 100° and h 40°, we obtain: $d = 19°\ .3$ and $t = 3^h\ 32^m$
2. t converted to arc = 53°
3. Long. 87° 48′ − 53° = 34° 48′ = G. H. A. (approximate) at observation
4. Enter N. A. at "Correction to be added to tabulated G. H. A. of stars" with G. C. T. $1^h\ 32^m$ and obtain 23° 3′ .8 = increase of G. H. A. since 0^h
5.

G. H. A. at observation	34° 48′
Increase since 0^h	23° 3′ .8
G. H. A. at 0^h	11° 44′ .2 W.

6. Look in N. A. "Stars" for May 1937, date 6, to find one with approximately d 19° 18′ N. and G. H. A. at 0^h 11° 44′ .2 W. The nearest combination found is d 19° 30′ .3 N. with G. H. A. 10° 13′ .3. This is close enough and identifies this star as Arcturus.

Time of Sun on Prime Vertical
(H. O. 211)

When declination is less than latitude and the same name,* the time sun will be due E. or W. may be found as follows:

Estimate G. C. T. when sun will be on Prime Vertical, E. or W. as you may desire.

Take d for this G. C. T. from N. A.

Estimate L and Lo in which ship will be at this G. C. T.

Find L. H. A. (t) of sun when on P. V. by the following:

$$\frac{\csc\ d}{\csc\ L} = \csc h$$

$$\frac{\sec\ h}{\sec\ d} = \csc t$$

Combine t with D. R. Lo to get G. H. A. of sun on P. V.

Use method for transit to find G. C. T. of this G. H. A.

This G. C. T. will be approximate time (as good as D. R. Lo) when sun will be on P. V. and longitude sight can best be made.

The advantage of taking the observation for longitude at this time is that any ordinary error in the latitude used will not affect the accuracy of the result.

* If $d > L$ and same name, body never crosses P.V.
If d & L are opposite names and
$d < L$ body crosses P.V. below horizon,
$d > L$ body never crosses P.V.

Required time of sun on P. V., 31 July, 1942, in afternoon, at D. R. position Lat. 41° 46′ N., Long. 86° 51′ W.

Estimated G. C. T. for above: 22^h on 31 July.

Declination for this: 18° 15′ .7 N.

By H. O. 211:

d	18° 15′.7 N	A 50404	B 2243
L	41° 46′.0 N	− A 17646	(from)
		A 32758	B 5430
t	68° 19′.0 W		A 3187
+ Lo	86° 51′.0 W		
G.H.A.	155° 10′.0 W		

G.C.T.	22^h	To find G.C.T. for this when G.H.A. =	155° 10′
			148° 26′
			6° 44′
Add	26^m	to increase G.H.A. by	6° 30′
			14′
Add	56^s	to increase G.H.A. by	14′
G.C.T.	$22^h\ 26^m\ 56^s$	Time of sun on P.V.	0

For +6 zone and wartime this became $5^h\ 26^m\ 56^s$ P.M.

Civil to Sidereal Time Conversion

On July 13, 1939, when local civil time is $9^h\ 3^m\ 30^s$ in longitude 85° 15′ W. ($5^h\ 41^m$), what is the local sidereal time?

G.S.T. of 0^h G.C.T., July 13 (N.A. p. 3)	19^h	19^m	$54^s.5$
"Reduction" (here an increase) for $5^h\ 41^m$ (Table VI, p. 289).	+		$56^s.0$
L.S.T. of 0^h L.C.T. July 13	19^h	20^m	$50^s.5$
Add L.C.T.	9^h	3^m	30^s
"Reduction" (here an increase) for $9^h\ 3^m\ 30^s$ (Table VI, p. 289)		1^m	$29^s.3$
	28^h	25^m	$49^s.8$
Reject	24^h		
L.S.T.	4^h	25^m	$49^s.8$

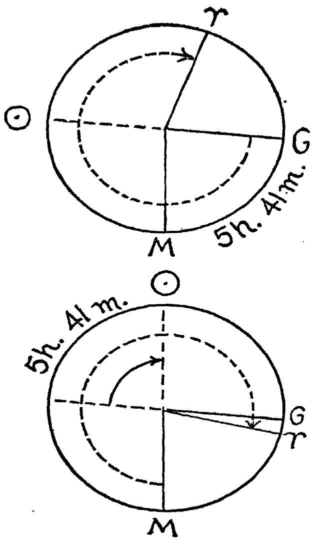
In case the above is not obvious to the student, a detailed explanation of each step with roughly accurate diagrams will be found below. Dotted curves represent sidereal time.

NOTE: In the N. A., Table VI and the table at bottom of pages 2 and 3 have two purposes:

1. Conversion of Civil to Sidereal (add)
2. Correction to G.S.T. for L.S.T.
   a. W. Longitude: add.
   b. E. Longitude: subtract.

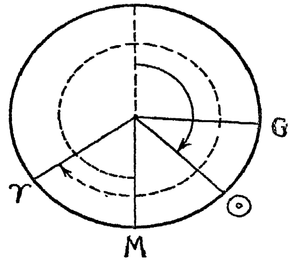
Starting with G.S.T. of 0^h G.C.T.,
To find L.S.T. of 0^h L.C.T.:—
For W. Long. add factor for Long. in Time from either table because L.S.T. is later = a passage of time into future.
For E. Long. subtract factor for Long. in Time from either table because L.S.T. is earlier = a passage of time into past.

G. S. T. of 0^h G. C. T. = 19^h 19^m 54^s. 5.

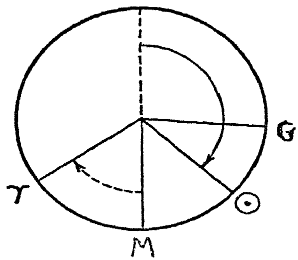


At 0^h L. C. T. we want the L. S. T. But then ☉ will have moved through 5^h 41^m of longitude since first diagram. (Angle between dotted meridians is same as angle between G and M meridians.) So ♈ will have also moved 5^h 41^m plus a certain amount. (Table VI.) This extra amount is because ♈ "gets around" faster than ☉ and for a given duration there are always more units of sidereal time than of civil time. For L. S. T. we start our dotted curve at local meridian M, thus discarding that part of dotted curve in first diagram which lies between G and M, or 5^h 41^m. Hence L. S. T. at 0^h L. C. T. is same as G. S. T. at 0^h G. C. T. plus "Reduction" of 56^s for the longitude. This totals 19^h 20^m 50^s .5.

But we must figure for the L. C. T., when ⊙ has moved $9^h\ 3^m\ 30^s$ since second diagram. Then ♈ will have also moved $9^h\ 3^m\ 30^s$ plus a certain amount. (Table VI.) So we extend the dotted curve $9^h\ 3^m\ 30^s + 1^m\ 29^s .3$. Adding this to L. S. T. of 0^h L. C. T. gives $28^h\ 25^m\ 49^s .8$.



As the last amount is over 24^h, and we do not count sidereal dates, we discard 24^h leaving L. S. T. = $4^h\ 25^m\ 49^s .8$ at L. C. T. $9^h\ 3^m$

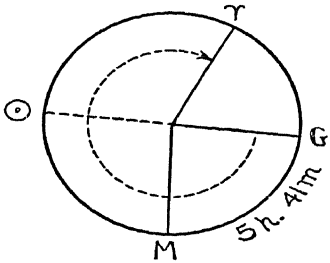


Sidereal to Civil Time Conversion

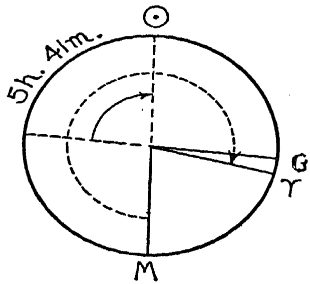
On July 13, 1939, when local sidereal time is $4^h\ 25^m\ 49^s .8$ in longitude 85° 15′ W. ($5^h\ 41^m$), what is the local civil time?

G.S.T. of 0^h G.C.T., July 13 (N.A. p. 3)	19^h	19^m	$54^s.5$
"Reduction" (here an increase) for $5^h\ 41^m$ (Table VI, p. 289) .	+		$56^s.0$
L.S.T. of 0^h L.C.T. July 13	19^h	20^m	$50^s.5$
L.S.T. given (+ 24^h for subtracting above)	28^h	25^m	$49^s.8$
Sidereal time interval since 0^h L.C.T.	9^h	4^m	$59^s.3$
Reduction (actual) for $9^h\ 4^m\ 59^s.3$ (Table V, p. 287)	−	1^m	$29^s.3$
L.C.T.	9^h	3^m	30^s

In case the above is not obvious to the student, a detailed explanation of each step with roughly accurate diagrams will be found below. Dotted curves represent sidereal time.

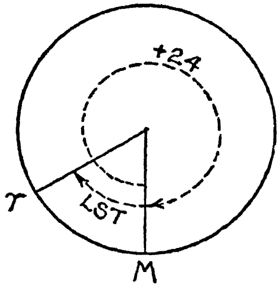


G. S. T. of 0^h G. C. T. = 19^h 19^m 54^s .5.

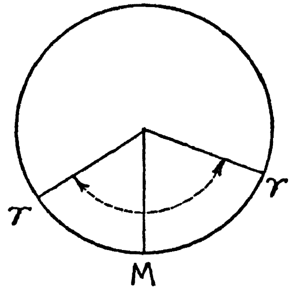


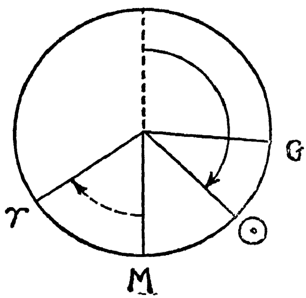
At 0^h L. C. T. we want the L. S. T. But then ⊙ will have moved through 5^h 41^m of longitude since first diagram. (Angle between dotted meridians is same as angle between G and M meridians.) So ♈ will have also moved 5^h 41^m plus a certain amount. (Table VI.) This extra amount is because ♈ "gets around" faster than ⊙ and for a given duration there are always more units of sidereal time than of civil time. For L. S. T. we start our dotted curve at local meridian M, thus discarding that part of dotted curve in first diagram which lies between G. and M., or 5^h 41^m. Hence L. S. T. at 0^h L. C. T. is same as G. S. T. at 0^h G. C. T. plus "Reduction" of 56^s for the longitude. This totals 19^h 20^m 50^s .5.

We know the L. S. T. at the unknown L. C. T. is $4^h\ 25^m\ 49^s .8$. We want to know the interval in sidereal time between this L. S. T. and the L. S. T. of 0^h L. C. T. as shown in second diagram. In order to subtract $19^h\ 20^m\ 50^s .5$ from $4^h\ 25^m\ 49^s .8$ we add 24^h to the latter. (Other combinations may not require this.)



This subtraction gives the desired interval in sidereal time between L. S. T. at the unknown L. C. T. and L. S. T. of 0^h L. C. T. It is $9^h\ 4^m\ 59^s .3$. This is a slightly larger quantity than the civil time for the same duration. To convert it to civil it must be reduced by an amount $1^m\ 29^s .3$ found for it in Table V. Subtracting this gives the interval now in civil time. It is $9^h\ 3^m\ 30^s$.





This shows L. C. T. $9^h\ 3^m\ 30^s$ at L. S. T. $4^h\ 25^m\ 49^s .8$.

A Uniform Method for More Exact Time of Local Transit of Any Body

Find G.H.A. (W. or E. to 180°) at 0^h G.C.T. of date on which local transit is desired and compare with Lo.

Rule I, for West Longitude: If G.H.A. (W.) at 0^h G.C.T. of date on which local transit is wanted is $<$ Lo. W., use G.H.A. in calculation as of one date more.

Rule II, for East Longitude: If G.H.A. (E.) at 0^h G.C.T. of date on which local transit is wanted is $<$ Lo. E., use G.H.A. in calculation as of one date less.

When neither rule applies, use G.H.A. in calculation as of date on which local transit is wanted.

When L.H.A. = 0° (transit), G.H.A. = Lo. (always expressed as W. to 360°).

Find G.H.A. of planet or star at 0^h G.C.T. of date as found above and express as W. or E. to 180°. Find G.H.A. of sun or moon on date as found above at whatever hour the G.H.A. is closest under Lo. W. to 360°. These are starting positions.

Calculate the angular distance to be timed from body's starting position, west to its position at transit, adding an east to a west angle if necessary.

The appropriate "Correction to G.H.A." table must now be used in each case (sun, moon, planet or star) to add up the total G.C.T. equal to this angular distance, using correct H. D. for moon or V. p. M. for planet. This gives G.C.T. of local transit.

Apply longitude in time, or zone description with reversed sign, to G.C.T. for L.C.T. or Z.T. of local transit.

Required: L. Tr. *Denebola*, June 3, 1939, Lo. 88° 30′ W.

(G.H.A. (W.) 6/3 0^h G.C.T. = 74° 03′.3 = <Lo. W.: Rule I)

When L.H.A. = 0°, G.H.A. = Lo.			88° 30′.0 W.
At G.C.T. 6/4 0^h, G.H.A. =			(−) 75° 02′.5 W.
			13° 27′.5 to go
N.A. p. 214:	0^h 53^m	represents	(−) 13° 17′.2
			10′.3 to go
	41^s		(−) 10′.3

G.C.T. 6/4		0^h 53^m 41^s	= G. Time of L. Tr.
Lo. W.	(−)	5^h 54^m 00^s	
L.C.T. 6/3		18^h 59^m 41^s	= L. Time of L. Tr.

Required: L. Tr. *Betelgeux*, Dec. 9, 1939, Lo. 90° E.

(G.H.A. (W.) 12/9 0^h G.C.T. = 348° 51′.0
from 350° 60′.0

G.H.A. (E.) = 11° 09′.0 = <Lo. E.: Rule II)

When L.H.A. = 0°, G.H.A. = Lo. = (360° − 90°) =			270° 00′.0 W.
At G.C.T. 12/8 0^h, G.H.A. = 347° 51′.9 W.			
from 359° 60′.0			
G.H.A. (E.)		12° 08′.1	(+) 12° 08′.1 E.
			282° 08′.1 to go
N.A. p. 216:	18^h 45^m	represents	(−) 282° 01′.2
			6′.9 to go
	$27^s.5$	"	(−) 6′.9

G.C.T. 12/8		18^h 45^m $27^s.5$	= G. Time of L. Tr.
Lo. E.	(+)	6^h 00^m $00^s.0$	
L.C.T. 12/9		0^h 45^m $27^s.5$	= L. Time of L. Tr.

Required: L. Tr. *Canopus*, Nov. 15, 1939, Lo. 24° 14′.0 W.

(G.H.A. (W.) 11/15 0^h G.C.T. = 317° 31′.2: No rule, use same date)

When L.H.A. = 0°, G.H.A. = Lo. =			24° 14′.0 W.
At G.C.T. 11/15 0^h, G.H.A. =	317° 31′.2 W.		
	from 359° 60′.0		
G.H.A. (E.) =	42° 28′.8		(+) 42° 28′.8 E.
			66° 42′.8 to go
N.A. p. 214:	4^h 26^m	represents	(−) 66° 40′.9
			1′.9 to go
"	$07^s.5$	"	(−) 1′.9
G.C.T. 11/15	4^h 26^m $07^s.5$ = G. Time of L. Tr.		0
Lo. W.	(−) 1^h 36^m $56^s.4$		
L.C.T. 11/15			

Required: L. Tr. *Aldebaran*, Dec. 20, 1939, Lo. 45° E.

(G.H.A. (W.) 12/20 0^h G.C.T. = 19° 32′.2: No rule, use same date)

When L.H.A. = 0°, G.H.A. = Lo. = (360° − 45°) =			315° 00′.0 W.
At G.C.T. 12/20 0^h, G.H.A. =			(−) 19° 33′.2 W.
			295° 26′.8 to go
N.A. p. 216:	19^h 38^m	represents	(−) 295° 18′.4
			8′.4 to go
	$33^s.5$	"	(−) 8′.4
G.C.T. 12/20	19^h 38^m $33^s.5$ = G. Time of L. Tr.		0
Lo. E.	(+) 3^h 00^m $00^s.0$		
L.C.T. 12/20	22^h 38^m $33^s.5$ = L. Time of L. Tr.		

Required: L. Tr. *Saturn*, June 1, 1939, Lo. 176° 19′.4 W.

(G.H.A. (W.) 6/1 0^h G.C.T. = 222° 41′.8: No rule, use same date)

When L.H.A. = 0°, G.H.A. = Lo. =			176° 19′.4 W.
At G.C.T. 6/1	0^h, G.H.A. = 222° 41′.8 W.		
	from 359° 60′.0		
G.H.A. (E.) =		137° 18′.2	(+) 137° 18′.2 E.
			313° 37′.6 to go
(H.A. Var. p. M. = 15′.0369)			
N.A. p. 159:	20^h	represents	(−) 300° 44′.4
			12° 53′.2 to go
" 161	51^m	"	(−) 12° 46′.8
			6′.4 to go
" "	$25^s.5$	"	(−) 6′.4
G.C.T. 6/1	20^h 51^m $25^s.5$ = G. Time of L. Tr.		0
Lo. W.	(−) 11^h 45^m $17^s.6$		
L.C.T. 6/1	9^h 06^m $07^s.9$ = L. Time of L. Tr.		

Required: L. Tr. *Moon*, Jan. 28, 1939, Lo. 15° 26′.3 W.

(G.H.A. (W.) 1/28 0^h G.C.T. = 99° 07′.9: No rule, use same date)

When L.H.A. = 0°, G.H.A. = Lo. =			15° 26′.3 W.
At G.C.T. 1/28	19^h	G.H.A.	(−) 14° 40′.2 W.
			46′.1 to go
(H.D. = 14° 29′.3)			
N.A. p. 133:	03^m	represents	(−) 43′.4
			2′.7 to go
	11^s	"	(−) 2′.7
G.C.T. 1/28	19^h 03^m $11^s.0$ = G. Time of L. Tr.		
Lo. W.	(−) 1^h 01^m $45^s.2$		
L.C.T. 1/28	18^h 01^m $25^s.8$ = L. Time of L. Tr.		

Remarks

Comparison of this method with that given in Dutton (7th. Ed., 1942, pp. 282–3) for Denebola shows the present procedure requires ten lines to Dutton's sixteen. My method for finding the date with which to enter the N. A. usually requires but one line (never more than three) whereas Dutton's requires seven. Bowditch (Revised Ed., 1938) does not explain how to pick the entering date. Neither Dutton nor Bowditch presents a uniform method for more exact time of local transit of any body.

Lack of space prevents adding rare sun problems where, near the 180th meridian, the longitude exceeds the sun's G.H.A. (W. or E.) at 0^h G.C.T.

Moon problems, on days when the moon is known to miss any transit, come out with a time of transit for the following day.

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Index